AGENT ADDA • JEE ADVANCED PHYSICS

ROTATIONAL MOTION

TEN CHAPTERS

50 challenging questions with visual cues, worked steps and physical checks.

Draw → choose the law → calculate → test the regime

ShunyaSaarthi in action: visual reasoning, guided discussion and step-by-step solutions.

SHUNYASAARTHI • ROHAN & SEEMA

TORQUE AND PIVOT CHOICE

Where a force acts matters

Torque measures turning effect about a chosen pivot. The perpendicular distance from pivot to force line is the lever arm; an axial force can have zero torque even when it is large. Assign clockwise and anticlockwise signs before adding moments.

$$\tau = rF\sin\theta = Fd$$

MISCONCEPTION

Force magnitude alone does not decide torque.

THIS CHAPTER'S PROBLEMS

01 Three forces on a hinged rod

02 Square plate and a diagonal force

03 Loaded beam with a cable

04 All zero-torque reference points

05 A fixed horizontal force on a turning rod

TORQUE AND PIVOT CHOICE

01

ROHAN & SEEMA / THINK TOGETHER

ROHAN
If P is huge, can it stop this rod turning?

SEEMA
First mark each force's line of action about O.

ROHAN
P points along the rod, so its lever arm is zero?

SEEMA
Exactly. Add only the other two signed torques.

PROBLEM 01 • THREE FORCES ON A HINGED ROD

A uniform rod of length L and finite moment of inertia is hinged at O. Two forces of magnitude F act at $x=L/3$ and $x=L$: the first makes 60° anticlockwise with the rod, the second 30° clockwise. A third force P acts along the rod at $x=L/2$. Find the signed net torque about O, then identify all P for which the rod has zero angular acceleration.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.

TORQUE AND PIVOT CHOICE

PROBLEM 01 • THREE FORCES ON A HINGED ROD

A uniform rod of length L and finite moment of inertia is hinged at O. Two forces of magnitude F act at $x=L/3$ and $x=L$: the first makes 60° anticlockwise with the rod, the second 30° clockwise. A third force P acts along the rod at $x=L/2$. Find the signed net torque about O, then identify all P for which the rod has zero angular acceleration.

DRAW FIRST

Draw O at the left, two force arrows at $L/3$ and L, and P directly along the rod. Mark the perpendicular force components.

1. Set anticlockwise positive: $\tau_1=(L/3)F\sin 60°=\sqrt{3}\,FL/6$; $\tau_2=-LF\sin 30°=-FL/2$.

2. The line of action of P passes through O, so $\tau_P=0$ for every P.

3. Thus $\tau_{net}=FL(\sqrt{3}-3)/6<0$. **No value of P** makes angular acceleration zero for $F,L>0$; an axial push cannot supply the missing torque. The rod's inertia is irrelevant to the zero-torque test.

EXAM PATTERN

Line of action → lever arm → signed torque.

MISCONCEPTION CHECK

Force magnitude alone does not decide torque.

Model assumptions and sign conventions are stated in the worked steps.

TORQUE AND PIVOT CHOICE

ROHAN & SEEMA / THINK TOGETHER

ROHAN
Could a stronger diagonal force balance the plate?

SEEMA
Extend that diagonal through the central pivot.

ROHAN
It passes through the pivot, whatever Q is!

SEEMA
Then calculate the moments of the other two forces.

PROBLEM 02 • SQUARE PLATE AND A DIAGONAL FORCE

A square plate of side a is pivoted at its centre. Three in-plane forces act: F upward at the right-upper corner, 2F leftward at the right-lower corner, and an unknown force of magnitude Q along the diagonal from the left-lower to the right-upper corner. Find the torque and determine whether Q can balance the first two forces.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.

TORQUE AND PIVOT CHOICE

02

PROBLEM 02 • SQUARE PLATE AND A DIAGONAL FORCE

A square plate of side a is pivoted at its centre. Three in-plane forces act: F upward at the right-upper corner, 2F leftward at the right-lower corner, and an unknown force of magnitude Q along the diagonal from the left-lower to the right-upper corner. Find the torque and determine whether Q can balance the first two forces.

DRAW FIRST

Centre at origin, corners $(\pm a/2, \pm a/2)$. Extend the unknown diagonal force's line through the centre.

1. At $(+a/2, +a/2)$, upward F gives $\tau_1 = +aF/2$.

2. At $(+a/2, -a/2)$, leftward 2F gives $\tau_2 = -aF$.

3. The diagonal force has zero moment about the centre whatever Q. Result $\tau = -aF/2$; **no Q can balance it**. Do not assume every adjustable force provides adjustable torque.

EXAM PATTERN

Line of action → lever arm → signed torque.

MISCONCEPTION CHECK

Force magnitude alone does not decide torque.

Model assumptions and sign conventions are stated in the worked steps.

TORQUE AND PIVOT CHOICE

ROHAN & SEEMA / THINK TOGETHER

ROHAN
The beam has two loads. Which force should we solve first?

SEEMA
Take moments about the hinge to remove its reactions.

ROHAN
Then tension is fixed by the vertical force component?

SEEMA
Yes. Use both force balances to find the hinge reaction.

PROBLEM 03 • LOADED BEAM WITH A CABLE

A nonuniform horizontal beam of length L has weight W acting at $x=0.7L$. It is hinged at $x=0$ and held by a cable attached at $x=L$, making angle θ above the beam. A downward point load P is at $x=0.4L$. Find cable tension and both components of hinge reaction; state the condition for a physically taut cable.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.

PROBLEM 03 • LOADED BEAM WITH A CABLE

A nonuniform horizontal beam of length L has weight W acting at $x=0.7L$. It is hinged at $x=0$ and held by a cable attached at $x=L$, making angle θ above the beam. A downward point load P is at $x=0.4L$. Find cable tension and both components of hinge reaction; state the condition for a physically taut cable.

DRAW FIRST

Put W at $0.7L$, P at $0.4L$, tension T at L, and hinge reactions H_x, H_y at 0.

1 Torque about the hinge: $TL\sin\theta=0.7WL+0.4PL$; hence $T=(0.7W+0.4P)/\sin\theta$.

2 Horizontal balance gives $H_x=-T\cos\theta$; vertical balance $H_y=W+P-T\sin\theta=0.3W+0.6P$.

3 A taut upward-supporting cable requires $\sin\theta>0$ for positive W, P. A cable angle near zero demands unbounded tension in this ideal model.

EXAM PATTERN

Line of action → lever arm → signed torque.

MISCONCEPTION CHECK

Force magnitude alone does not decide torque.

Model assumptions and sign conventions are stated in the worked steps.

TORQUE AND PIVOT CHOICE

ROHAN & SEEMA / THINK TOGETHER

ROHAN
Where can I move the pivot so this force makes no torque?

SEEMA
Put it anywhere on the force's line of action.

ROHAN
So a zero moment doesn't mean zero force?

SEEMA
Correct. Translation can still change.

PROBLEM 04 • ALL ZERO-TORQUE REFERENCE POINTS

A force $F=(F_x,F_y)$ acts at (a,b) on a lamina in the xy plane. Find every point (x_0,y_0) about which its torque vanishes. Describe geometrically why choosing a point on that set simplifies an equation of motion.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.

TORQUE AND PIVOT CHOICE

PROBLEM 04 • ALL ZERO-TORQUE REFERENCE POINTS

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DRAW FIRST

Draw the line of action of F through (a,b), then place O anywhere on that same line.

1. About (x_0,y_0), $\tau_z=(a-x_0)F_y-(b-y_0)F_x$.

2. Set $\tau_z=0$: $(a-x_0)F_y=(b-y_0)F_x$. For a nonzero force this is precisely its entire line of action.

3. This pivot removes F's torque from an equation, although the force still affects translation. If F=0, every point works.

EXAM PATTERN

Line of action → lever arm → signed torque.

MISCONCEPTION CHECK

Force magnitude alone does not decide torque.

Model assumptions and sign conventions are stated in the worked steps.

TORQUE AND PIVOT CHOICE

ROHAN & SEEMA / THINK TOGETHER

ROHAN
The push stays horizontal. Why does the torque change?

SEEMA
The perpendicular distance changes as the rod turns.

ROHAN
It becomes zero when the rod lies along the push?

SEEMA
Yes. Write the position cross the force with a sign.

PROBLEM 05 • A FIXED HORIZONTAL FORCE ON A TURNING ROD

A horizontal rod of length L is pivoted at its left end and initially held at rest. A constant horizontal force F directed rightward acts at its right end while the rod forms angle φ with the positive horizontal. Find $\tau(\varphi)$, its sign and all positions where it vanishes; explain why a constant applied force need not give constant angular acceleration.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.

TORQUE AND PIVOT CHOICE

PROBLEM 05 • A FIXED HORIZONTAL FORCE ON A TURNING ROD

A horizontal rod of length L is pivoted at its left end and initially held at rest. A constant horizontal force F directed rightward acts at its right end while the rod forms angle φ with the positive horizontal. Find $\tau(\varphi)$, its sign and all positions where it vanishes; explain why a constant applied force need not give constant angular acceleration.

DRAW FIRST

At rod angle φ, resolve its endpoint position perpendicular to rightward F.

1. $r=(L\cos\varphi, L\sin\varphi)$, $F=(F,0)$.

2. $\tau_z = r_x F_y - r_y F_x = -FL\sin\varphi$. It vanishes at $\varphi = n\pi$ and changes sign across these positions.

3. For a rigid rod with fixed-axis inertia I, $\alpha(\varphi) = -(FL/I)\sin\varphi$: constant force is not constant torque when its lever arm changes.

EXAM PATTERN

Line of action → lever arm → signed torque.

MISCONCEPTION CHECK

Force magnitude alone does not decide torque.

Model assumptions and sign conventions are stated in the worked steps.

ANGULAR LANGUAGE AND MOTION

Read rotation as motion

One rigid wheel gives every point the same angular displacement, speed and acceleration. Linear speed grows with radius. Angular acceleration changes tangential speed; angular speed also produces inward centripetal acceleration.

$\omega = d\theta/dt$; $\alpha = d\omega/dt$

MISCONCEPTION

Points on one wheel share ω, not linear speed.

THIS CHAPTER'S PROBLEMS

06 A changing angular acceleration

07 Points at R and 2R

08 Cubic angular displacement

09 Two reversing wheels

10 Measured acceleration direction

ANGULAR LANGUAGE AND MOTION

ROHAN & SEEMA / THINK TOGETHER

ROHAN
Where is angular speed greatest on this interval?

SEEMA
Integrate acceleration, then inspect the whole interval.

ROHAN
The positive peak might not be the largest speed?

SEEMA
Right. Compare absolute values, including the endpoint.

PROBLEM 06 • A CHANGING ANGULAR ACCELERATION

A wheel starts from rest with angular acceleration $\alpha(t)=at-bt^2$ for $0\leq t\leq 2a/b$, where $a,b>0$. Find every instant of maximum angular speed, total angular displacement, and the intervals in which its angular acceleration opposes its angular velocity.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.

PROBLEM 06 • A CHANGING ANGULAR ACCELERATION

A wheel starts from rest with angular acceleration $\alpha(t)=at-bt^2$ for $0\le t\le 2a/b$, where $a,b>0$. Find every instant of maximum angular speed, total angular displacement, and the intervals in which its angular acceleration opposes its angular velocity.

DRAW FIRST

Sketch $\alpha(t)=t(a-bt)$, then $\omega(t)$; mark $t=a/b$ and $t=3a/(2b)$.

1. With $\omega(0)=0$, integrate: $\omega=at^2/2-bt^3/3=t^2(a/2-bt/3)$.

2. ω rises to a local maximum $a^3/(6b^2)$ at $t=a/b$, crosses zero at $3a/(2b)$, then falls to $-2a^3/(3b^2)$ at $2a/b$. The **largest angular speed** $|\omega|$ on the stated closed interval occurs at the final endpoint; distinguish this from maximum signed velocity.

3. Integrate again: $\theta-\theta_0=at^3/6-bt^4/12$. At $t=2a/b$, net angular displacement is zero, despite a nonzero traveled angle.

4. α opposes nonzero ω only for $a/b<t<3a/(2b)$.

CHECK THE MODEL

Points on one wheel share ω, not linear speed.

Model assumptions and sign conventions are stated in the worked steps.

ANGULAR LANGUAGE AND MOTION

ROHAN & SEEMA / THINK TOGETHER

ROHAN
Do the inner and outer marks have the same acceleration?

SEEMA
They share angular quantities, but each linear part scales with radius.

ROHAN
Can acceleration be normal to velocity here?

SEEMA
Only if the tangential component vanishes.

PROBLEM 07 • POINTS AT R AND 2R

Two coaxial particles are fixed at radii R and 2R on a rigid wheel. At an instant the wheel has ω and α of opposite signs. Compare their tangential and centripetal acceleration vectors and determine when their total accelerations are perpendicular to their velocities.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.

ANGULAR LANGUAGE AND MOTION

PROBLEM 07 • POINTS AT R AND 2R

Two coaxial particles are fixed at radii R and 2R on a rigid wheel. At an instant the wheel has ω and α of opposite signs. Compare their tangential and centripetal acceleration vectors and determine when their total accelerations are perpendicular to their velocities.

DRAW FIRST

At each point draw inward $a_c=\omega^2 r$ and tangential $a_t=\alpha r$; the farther pair is twice as long.

1. Tangential magnitudes are $|\alpha|R$ and $2|\alpha|R$; radial magnitudes $\omega^2 R$ and $2\omega^2 R$. Corresponding acceleration vectors are parallel and the outer vector is twice the inner.

2. Velocity is tangential. Total acceleration is perpendicular to velocity only if its tangential part vanishes, $\alpha=0$ (or the point is at $r=0$). The given nonzero opposite-signed ω,α do not meet this condition.

EXAM PATTERN

Integrate α to ω, then ω to θ; track reversals.

MISCONCEPTION CHECK

Points on one wheel share ω, not linear speed.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER

ROHAN
The wheel stops again. Has the mark returned to its start?

SEEMA
Find the angle at that time; stopping is not returning.

ROHAN
And its radial acceleration is then zero?

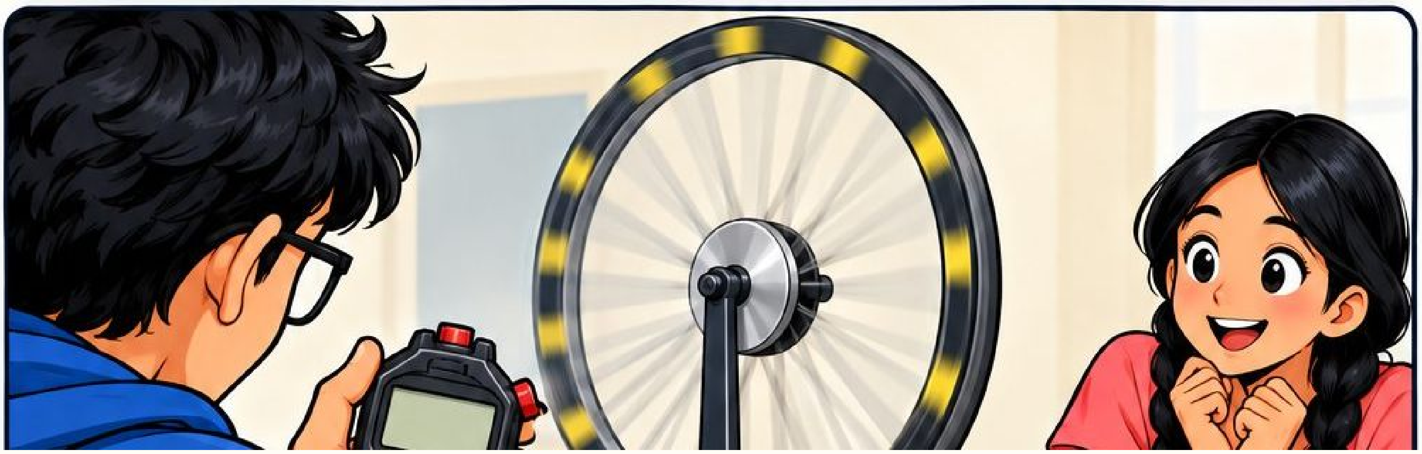
SEEMA
Yes. The tangential part can still remain.

PROBLEM 08 • CUBIC ANGULAR DISPLACEMENT

A disc of radius R rotates with $\theta(t)=At^3-Bt^2$, $A,B>0$. A point initially at the positive x axis is observed when its angular velocity first returns to zero after $t=0$. Find its position modulo 2π, total path length traveled by that point, and its instantaneous acceleration.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



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A disc of radius R rotates with $\theta(t)=At^3-Bt^2$, $A,B>0$. A point initially at the positive x axis is observed when its angular velocity first returns to zero after $t=0$. Find its position modulo 2π, total path length traveled by that point, and its instantaneous acceleration.

DRAW FIRST

Draw $\theta(t)$, decreasing from zero to its minimum before the marked instant.

1. $\omega=3At^2-2Bt=t(3At-2B)$; the first later zero is $t^*=2B/(3A)$.

2. $\theta(t^*)=-4B^3/(27A^2)$, so the point is at polar angle $-4B^3/(27A^2)$ modulo 2π and has traveled distance $4RB^3/(27A^2)$.

3. $\alpha(t^*)=6At^*-2B=2B$. Since $\omega(t^*)=0$, centripetal acceleration vanishes and the instantaneous acceleration is purely tangential of magnitude $2BR$, in the positive rotation direction.

EXAM PATTERN

Integrate α to ω, then ω to θ; track reversals.

MISCONCEPTION CHECK

Points on one wheel share ω, not linear speed.

Model assumptions and sign conventions are stated in the worked steps.

ANGULAR LANGUAGE AND MOTION

ROHAN & SEEMA / THINK TOGETHER

ROHAN
How do we count meetings after the wheels reverse?

SEEMA
Use unwrapped angles and set their difference to 2πn.

ROHAN
Should we keep every quadratic root?

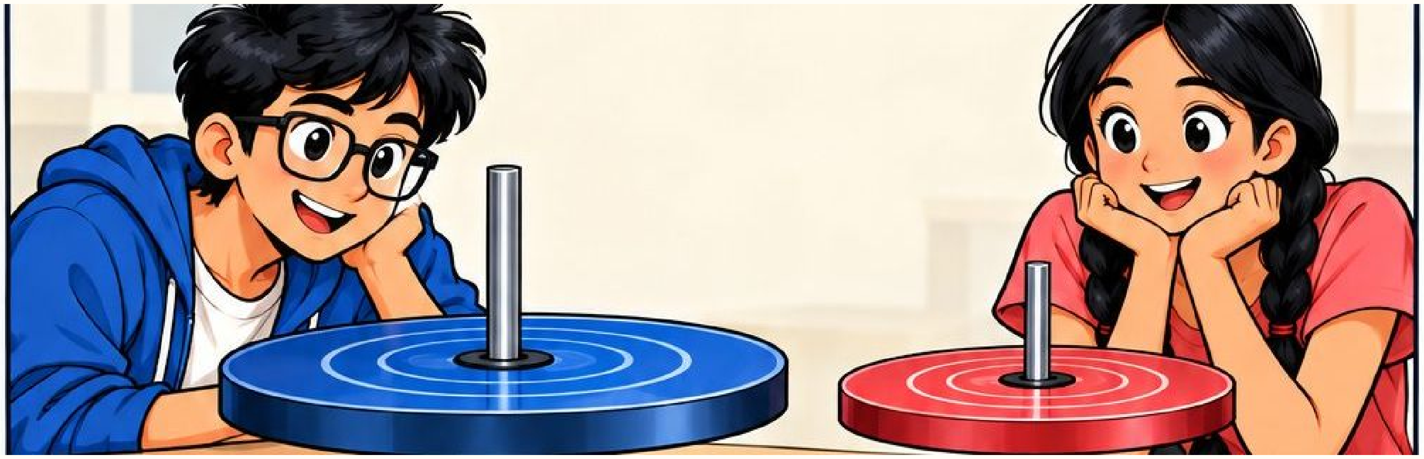
SEEMA
Keep only real, positive times, then mark the reversals.

PROBLEM 09 • TWO REVERSING WHEELS

Two wheels begin with angular velocities $+\omega$ and -2ω, and constant angular accelerations $-\alpha$ and $+\alpha/2$ respectively, where $\omega,\alpha>0$. They carry painted marks that coincide at $t=0$. Find all later coincidence times and identify any repeated coincidences after either wheel reverses.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



PROBLEM 09 • TWO REVERSING WHEELS

Two wheels begin with angular velocities $+\omega$ and -2ω, and constant angular accelerations $-\alpha$ and $+\alpha/2$ respectively, where $\omega,\alpha>0$. They carry painted marks that coincide at $t=0$. Find all later coincidence times and identify any repeated coincidences after either wheel reverses.

DRAW FIRST

Plot their angular positions unwrapped; coincidences occur when their difference crosses an integer multiple of 2π.

1. $\theta_A=\omega t-\alpha t^2/2$; $\theta_B=-2\omega t+\alpha t^2/4$. Thus $\Delta\theta=3\omega t-3\alpha t^2/4$.

2. For each integer n solve $\Delta\theta=2\pi n$: $3\alpha t^2/4-3\omega t+2\pi n=0$; retain real roots $t>0$. Explicitly $t=[3\omega\pm\sqrt{9\omega^2-6\pi\alpha n}]/(3\alpha/2)$. For $n<0$ only the plus branch is positive.

3. A reverses at ω/α, B at $4\omega/\alpha$. Evaluate the retained roots against these times to classify coincidences. For $n=0$, besides $t=0$ they coincide again at $t=4\omega/\alpha$, exactly when B reverses; later $n<0$ solutions occur after that.

EXAM PATTERN

Integrate α to ω, then ω to θ; track reversals.

MISCONCEPTION CHECK

Points on one wheel share ω, not linear speed.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER

ROHAN
The acceleration arrow is tilted. Can we find the spin?

SEEMA
Resolve it inward and tangentially.

ROHAN
Will that tell us clockwise or anticlockwise?

SEEMA
Not from acceleration alone; note the sign ambiguity.

PROBLEM 10 • MEASURED ACCELERATION DIRECTION

A bead fixed to a turntable at radius r has measured acceleration magnitude A and its acceleration makes angle β with the inward radial direction, $0<\beta<\pi/2$. Determine the possible instantaneous angular speed and magnitude of angular acceleration. Discuss what the measurement cannot reveal about their signs.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.

PROBLEM 10 • MEASURED ACCELERATION DIRECTION

A bead fixed to a turntable at radius r has measured acceleration magnitude A and its acceleration makes angle β with the inward radial direction, $0<\beta<\pi/2$. Determine the possible instantaneous angular speed and magnitude of angular acceleration. Discuss what the measurement cannot reveal about their signs.

DRAW FIRST

Resolve the observed A into inward $A\cos\beta$ and tangential $A\sin\beta$.

1. $r\omega^2=A\cos\beta$, so $|\omega|=\sqrt{A\cos\beta/r}$.

2. $r|\alpha|=A\sin\beta$, so $|\alpha|=A\sin\beta/r$.

3. Acceleration alone cannot distinguish clockwise from anticlockwise rotation. If the tangential side of the inward direction is known, it fixes α's direction, but not ω's sign.

EXAM PATTERN

Integrate α to ω, then ω to θ; track reversals.

MISCONCEPTION CHECK

Points on one wheel share ω, not linear speed.

Model assumptions and sign conventions are stated in the worked steps.

MOMENT OF INERTIA

Mass distribution controls response

Moment of inertia weights each mass element by the square of its distance from the axis. Move mass outward and the same torque gives less angular acceleration. Build composite objects by addition and shift parallel axes carefully.

$I = \Sigma mr^2$; $I = I_{cm} + Md^2$

MISCONCEPTION

Equal mass does not imply equal I.

THIS CHAPTER'S PROBLEMS

11 Remove and relocate a ring arc

12 Off-centre hole in a disc

13 Four masses and the least-inertia line

14 A bent uniform rod

15 Two joined discs

ROHAN & SEEMA / THINK TOGETHER

ROHAN
If mass stays on the rim, does the ring's inertia change?

SEEMA
About the original centre, every moved bit remains at R.

ROHAN
But the centre of mass shifts left?

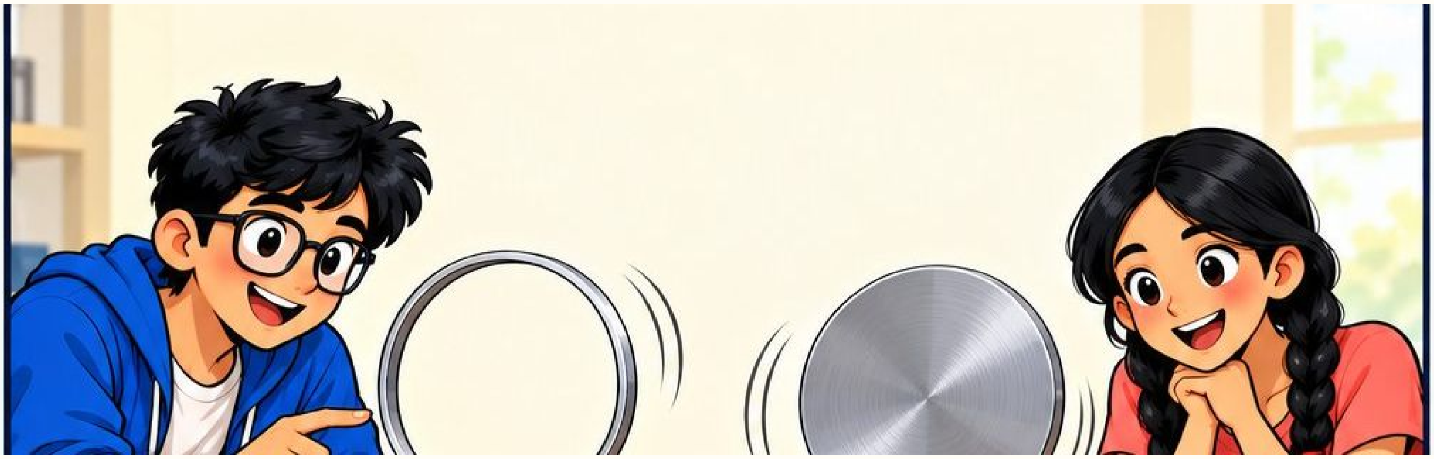
SEEMA
Yes. Average the removed arc before relocating its mass.

PROBLEM 11 • REMOVE AND RELOCATE A RING ARC

A thin circular ring of mass M, radius R, has a small segment of angular width 2φ centred at angle zero removed. The removed part is then attached as a point mass at the diametrically opposite rim point. Find the new centre of mass and moment of inertia about the original ring centre. Take the ring's original uniform linear density and use the removed arc's exact mass.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



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DRAW FIRST

Ring centre O, removed arc symmetric about +x, replacement point at $(-R,0)$. The arc's centre is not on the rim.

1. Removed mass $\mu=M\varphi/\pi$. Its centre is at $x=R\sin\varphi/\varphi$, since averaging $R\cos\psi$ over $-\varphi\le\psi\le\varphi$ gives $R\sin\varphi/\varphi$.

2. Original ring has zero first moment. New $x_{CM}=[-\mu R-\mu R\sin\varphi/\varphi]/M=-R(\varphi+\sin\varphi)/\pi$; $y_{CM}=0$.

3. Every removed bit and the replacement point lie at distance R from O, so I_O remains MR^2. If an axis through the *new* CM is requested, subtract Mx_{CM}^2 by the parallel-axis theorem.

CHECK THE MODEL

Equal mass does not imply equal I.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER

ROHAN
Can I subtract just the hole's central-disc inertia?

SEEMA
Include the hole's offset from the original axis too.

ROHAN
Then shift the remaining plate to its new centre?

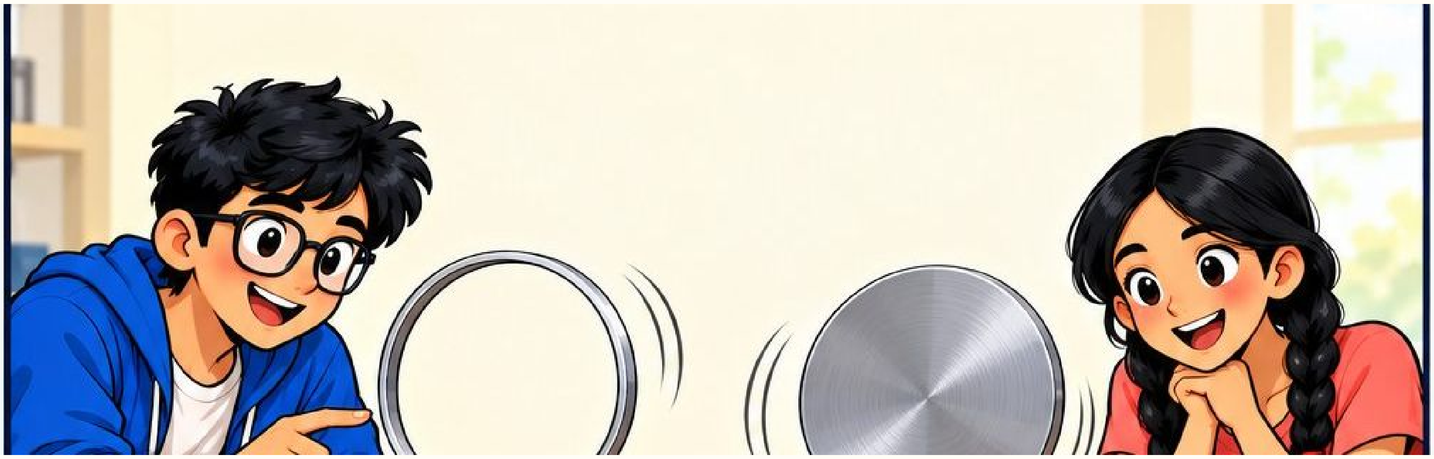
SEEMA
Exactly. Use the parallel-axis theorem twice.

PROBLEM 12 • OFF-CENTRE HOLE IN A DISC

A uniform disc of radius R has a circular hole of radius R/3 cut with its centre R/2 from the disc's centre. Determine the remaining object's centre of mass and I about an axis normal to the disc through that centre of mass, in terms of the original disc mass M.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



PROBLEM 12 • OFF-CENTRE HOLE IN A DISC

A uniform disc of radius R has a circular hole of radius $R/3$ cut with its centre $R/2$ from the disc's centre. Determine the remaining object's centre of mass and I about an axis normal to the disc through that centre of mass, in terms of the original disc mass M.

DRAW FIRST

Treat the hole as a negative disc of mass $M/9$ at $x=R/2$.

1. Remaining mass $M'=8M/9$. Its centre is $x_{CM}=-[(M/9)(R/2)]/(8M/9)=-R/16$.

2. About the original centre, $I_O=(1/2)MR^2-[(1/2)(M/9)(R/3)^2+(M/9)(R/2)^2]=(151/324)MR^2$.

3. Shift from O to the remaining object's CM: $I_{CM}=I_O-M'(R/16)^2=(1199/2592)MR^2$. Both the missing disc's intrinsic I and its offset are essential.

EXAM PATTERN

Build I from parts, then shift to the target axis.

MISCONCEPTION CHECK

Equal mass does not imply equal I.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER

ROHAN
Which line through the masses has least inertia?

SEEMA
Find their weighted centre, then the principal axes.

ROHAN
A coordinate axis may not be the best one?

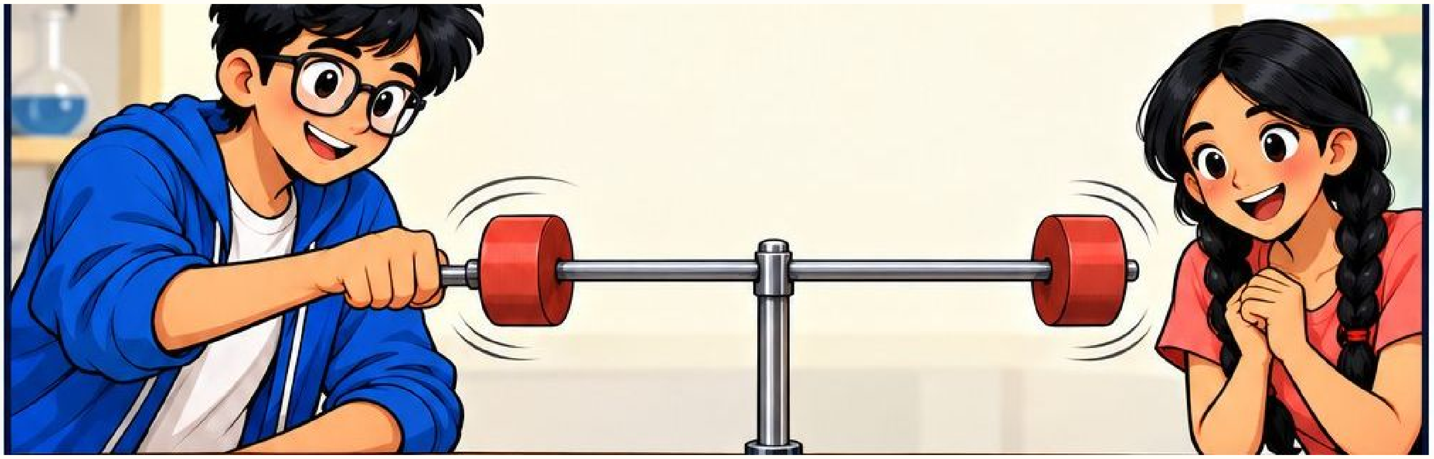
SEEMA
Right. Diagonalize the in-plane inertia matrix.

PROBLEM 13 • FOUR MASSES AND THE LEAST-INERTIA LINE

Four point masses m,2m,3m,4m occupy (a,0),(0,a),(−a,0),(0,−a). Find the centroid, I about the z axis through the centroid, and the in-plane direction of a line through the centroid that minimizes I about that line.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



PROBLEM 13 • FOUR MASSES AND THE LEAST-INERTIA LINE

Four point masses $m, 2m, 3m, 4m$ occupy $(a,0), (0,a), (-a,0), (0,-a)$. Find the centroid, I about the z axis through the centroid, and the in-plane direction of a line through the centroid that minimizes I about that line.

DRAW FIRST

Plot the four masses on coordinate axes. The optimal in-plane axis passes through the weighted centroid, not the origin.

1. Total mass $10m$ and centroid $C=(-a/5,-a/5)$.

2. By subtracting the centroid shift from Σmr^2, $I_{z,C}=(\Sigma m(x^2+y^2))-10m(2a^2/25)=10ma^2-(4/5)ma^2=(46/5)ma^2$.

3. In-plane axis inertia matrix through C is $(ma^2/5)$ [[28,2],[2,18]]. Its smaller eigenvalue is $I_{min}=(23-\sqrt{29})ma^2/5$.

4. A minimum-axis direction (u_x,u_y) obeys $u_y/u_x=-(5+\sqrt{29})/2$. The perpendicular principal line gives the larger eigenvalue; their sum equals $I_{z,C}$.

CHECK THE MODEL

Equal mass does not imply equal I.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER

ROHAN
The bent rod still has length L. Is I unchanged?

SEEMA
No. Mark the two arm masses and their centroids.

ROHAN
Compute around the joint before shifting axes?

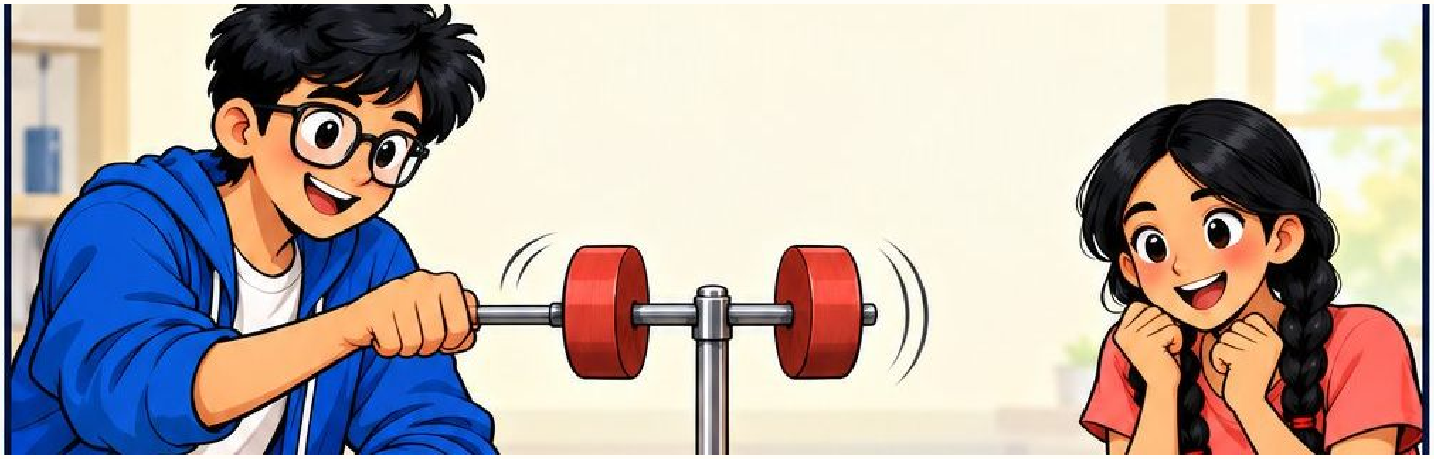
SEEMA
That's the cleanest route to I about the centre.

PROBLEM 14 • A BENT UNIFORM ROD

A uniform rod of mass M and length L is bent without stretching into two perpendicular arms of lengths L/3 and 2L/3 joined at one end. Find its centre of mass and I about an axis perpendicular to the plane through the centre of mass.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



PROBLEM 14 • A BENT UNIFORM ROD

A uniform rod of mass M and length L is bent without stretching into two perpendicular arms of lengths $L/3$ and $2L/3$ joined at one end. Find its centre of mass and I about an axis perpendicular to the plane through the centre of mass.

DRAW FIRST

Put the common joint at (0,0); shorter arm along +x and longer along +y.

1. The arms have masses $M/3$ and $2M/3$, with centres $(L/6,0)$ and $(0,L/3)$. Therefore $C=(L/18,2L/9)$.
2. About the joint, $I_O=(M/L)[(L/3)^3+(2L/3)^3]/3=ML^2/9$.
3. Shift to C: $I_C=I_O-M[(L/18)^2+(2L/9)^2]=(19/324)ML^2$.

EXAM PATTERN

Build I from parts, then shift to the target axis.

MISCONCEPTION CHECK

Equal mass does not imply equal I.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER

ROHAN
Do I treat the joined discs as one big disc?

SEEMA
Add each disc's own I and its centre offset.

ROHAN
Then move from their midpoint to the tangent axis?

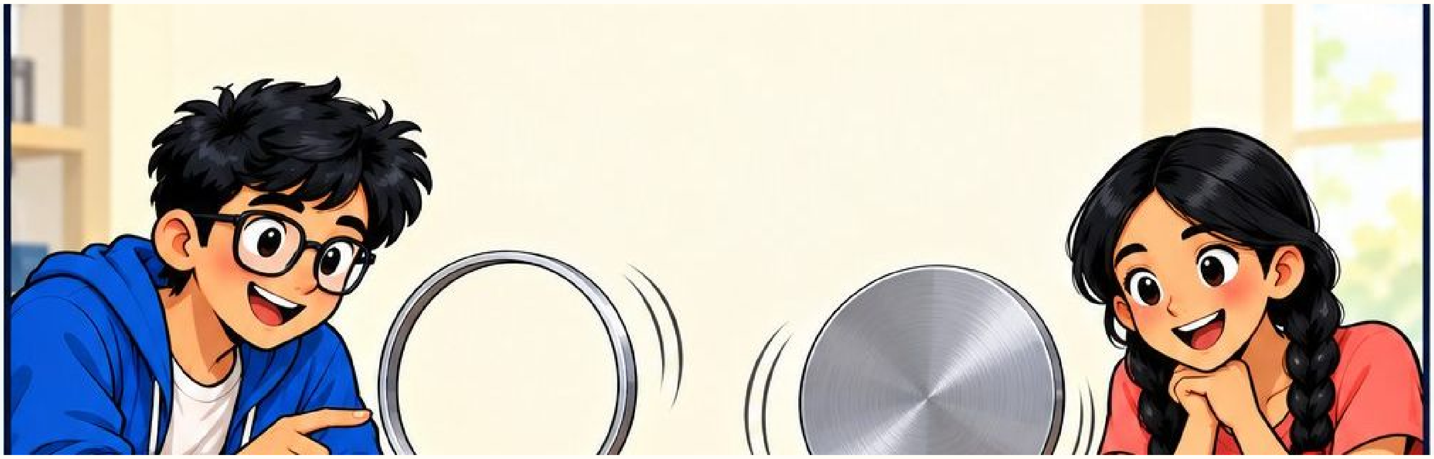
SEEMA
Yes, using the combined mass for that shift.

PROBLEM 15 • TWO JOINED DISCS

Two identical solid discs, each M and R, are rigidly joined with parallel axes and centre separation 2R. Find I about an axis perpendicular to their plane through the combined centre of mass and about a parallel tangent axis to the outer edge. Explain which parallel-axis shifts are necessary.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



PROBLEM 15 • TWO JOINED DISCS

Two identical solid discs, each M and R, are rigidly joined with parallel axes and centre separation 2R. Find I about an axis perpendicular to their plane through the combined centre of mass and about a parallel tangent axis to the outer edge. Explain which parallel-axis shifts are necessary.

DRAW FIRST

Centres lie at $x=\pm R$; the combined centre is midway. The farthest tangent line is $x=2R$.

1. Each disc contributes $I_{own}=(1/2)MR^2$ and MR^2 from its centre's offset. Hence $I_{mid}=2[(1/2)MR^2+MR^2]=3MR^2$.

2. A parallel tangent axis 2R from the combined centre has $I_{tangent}=I_{mid}+(2M)(2R)^2=11MR^2$. Use the *total* mass for the second shift.

EXAM PATTERN

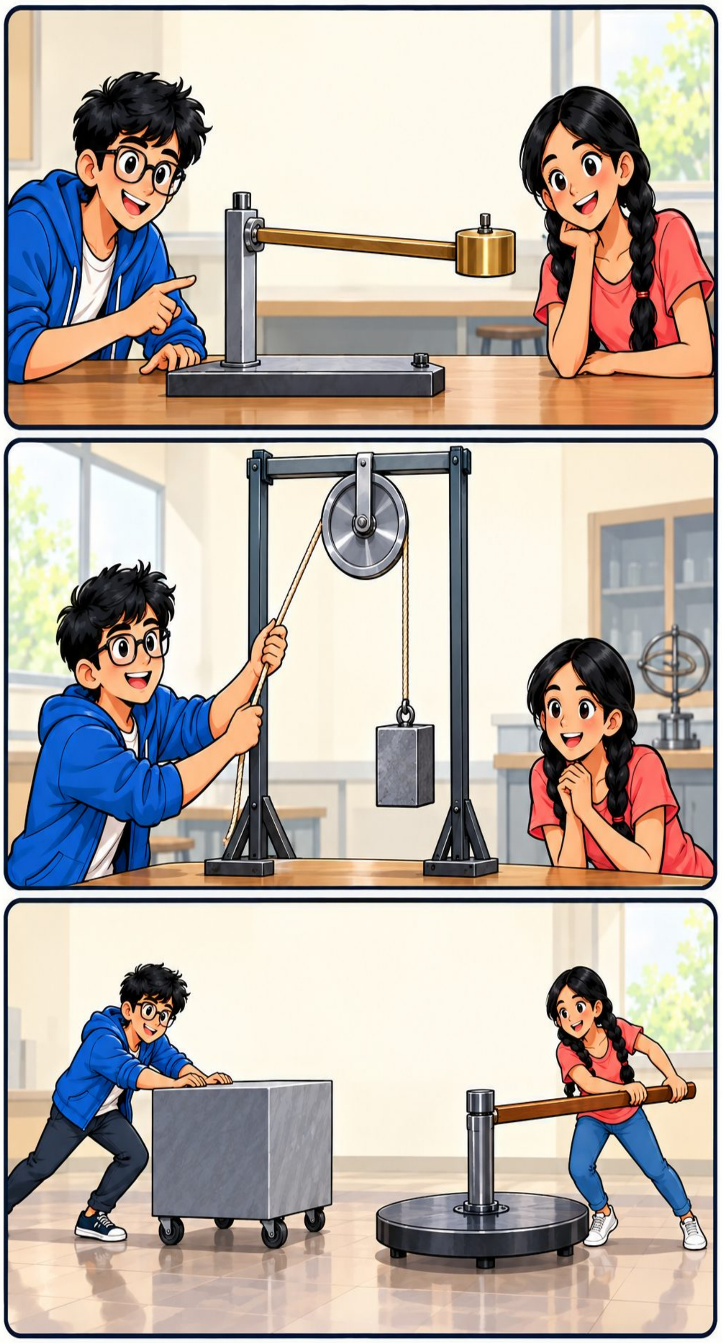
Build I from parts, then shift to the target axis.

MISCONCEPTION CHECK

Equal mass does not imply equal I.

Model assumptions and sign conventions are stated in the worked steps.

TORQUE AND ANGULAR ACCELERATION



Link forces to angular acceleration

A free-body diagram lists actual forces, not their components as extra forces. For a fixed axis, net external torque controls angular acceleration. Translation, rotation and a string or contact constraint usually must be solved together.

$\Sigma\tau = I\alpha$ (fixed axis)

MISCONCEPTION

A force can have zero moment about one axis.

THIS CHAPTER'S PROBLEMS

16 Rod falling from horizontal

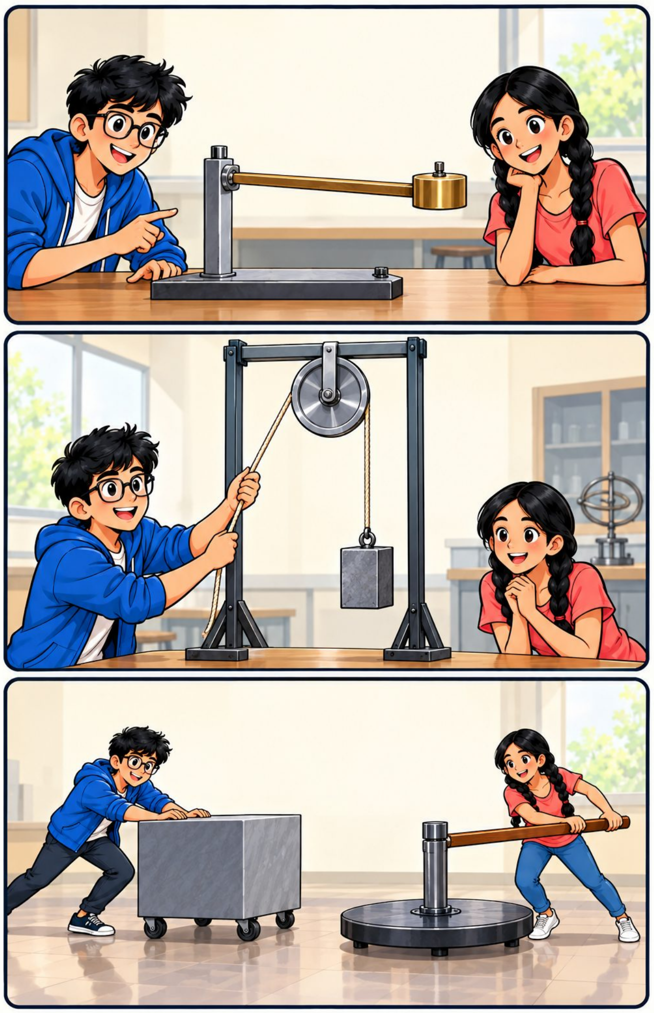
17 Speed-dependent driving torque

18 Hanging mass and a fixed-axle cylinder

19 Off-centre pin and near-end force

20 A brake with torque proportional to angle

ROHAN & SEEMA / THINK TOGETHER



ROHAN
At release, what acceleration does the rod's centre have?

SEEMA
First get angular acceleration from weight's hinge torque.

ROHAN
There is no centripetal part while ω is zero?

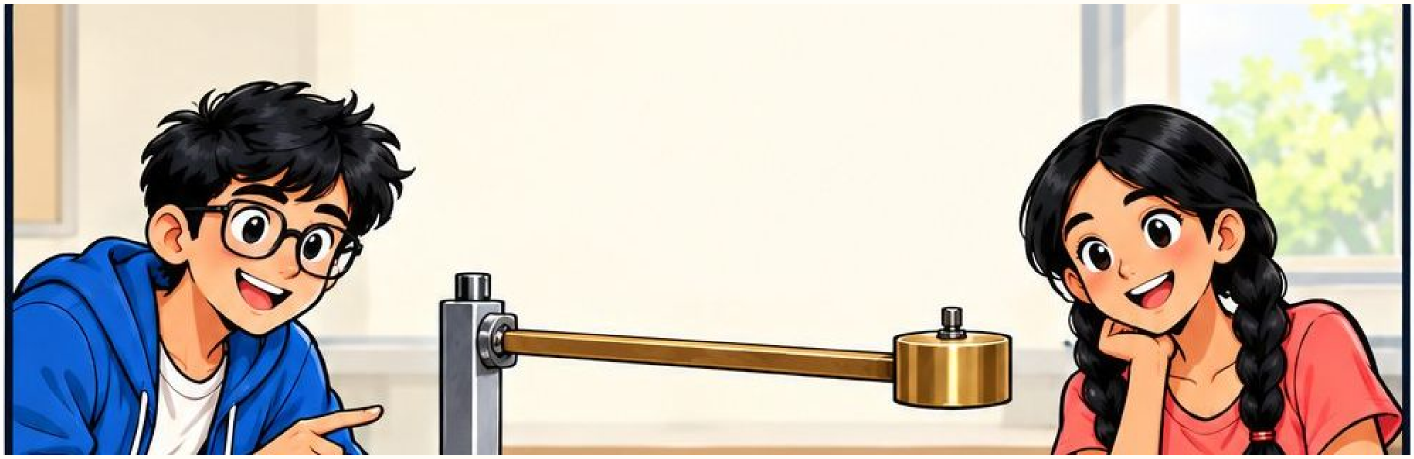
SEEMA
Correct. Then Newton's law gives the hinge force.

PROBLEM 16 • ROD FALLING FROM HORIZONTAL

A uniform rod of length L and mass M is hinged at one end and released from rest from horizontal. Find initial angular acceleration and hinge reaction components immediately after release. Then obtain its angular speed when vertical using energy.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



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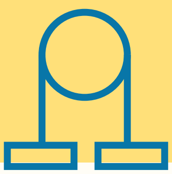


DRAW FIRST

Draw the centre at L/2, weight straight down and hinge at one end. At release $\omega=0$, so no radial acceleration yet.

1. $I_{hinge}=ML^2/3$. Initial torque $MgL/2$ gives $\alpha_0=3g/(2L)$, rotating downward.

2. The centre initially accelerates downward by $\alpha_0 L/2=3g/4$. Newton's vertical equation $H_y-Mg=-3Mg/4$ gives $H_y=Mg/4$; $H_x=0$.

3. At the bottom the centre has fallen L/2. Energy: $MgL/2=(1/2)(ML^2/3)\omega^2$; therefore $\omega=\sqrt{3g/L}$. The hinge force at the bottom is a different quantity from its initial value.

EXAM PATTERN

Draw each body; link ΣF to $\Sigma\tau$ with constraints.

MISCONCEPTION CHECK

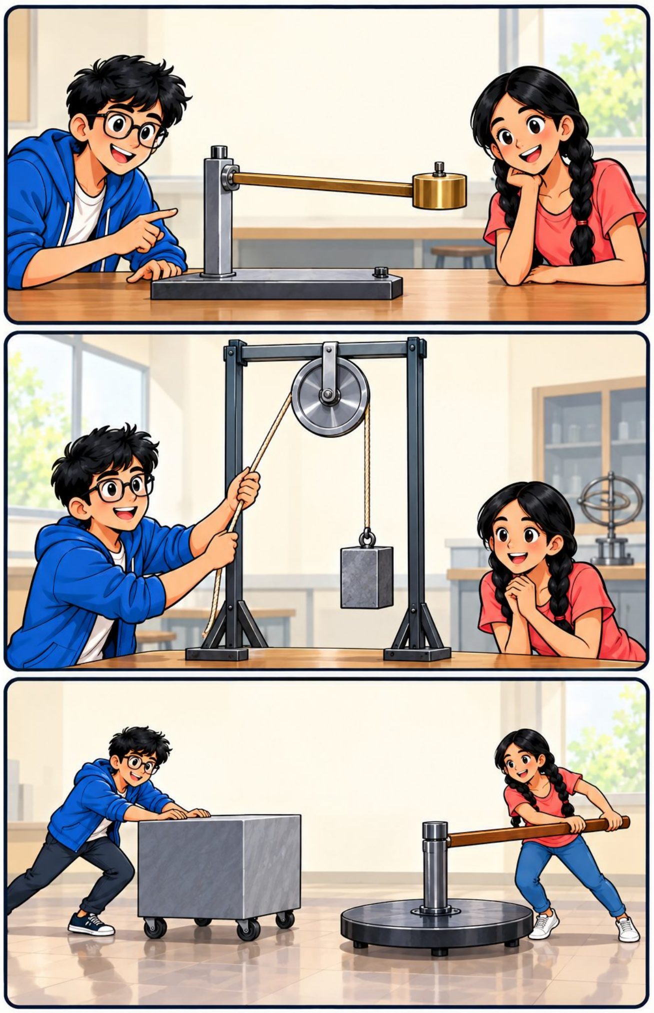
A force can have zero moment about one axis.

Model assumptions and sign conventions are stated in the worked steps.

TORQUE AND ANGULAR ACCELERATION

17

ROHAN & SEEMA / THINK TOGETHER



ROHAN
The motor weakens as the wheel speeds up. Where does it settle?

SEEMA
Set drive torque equal to resisting torque.

ROHAN
How do we find the half-speed time?

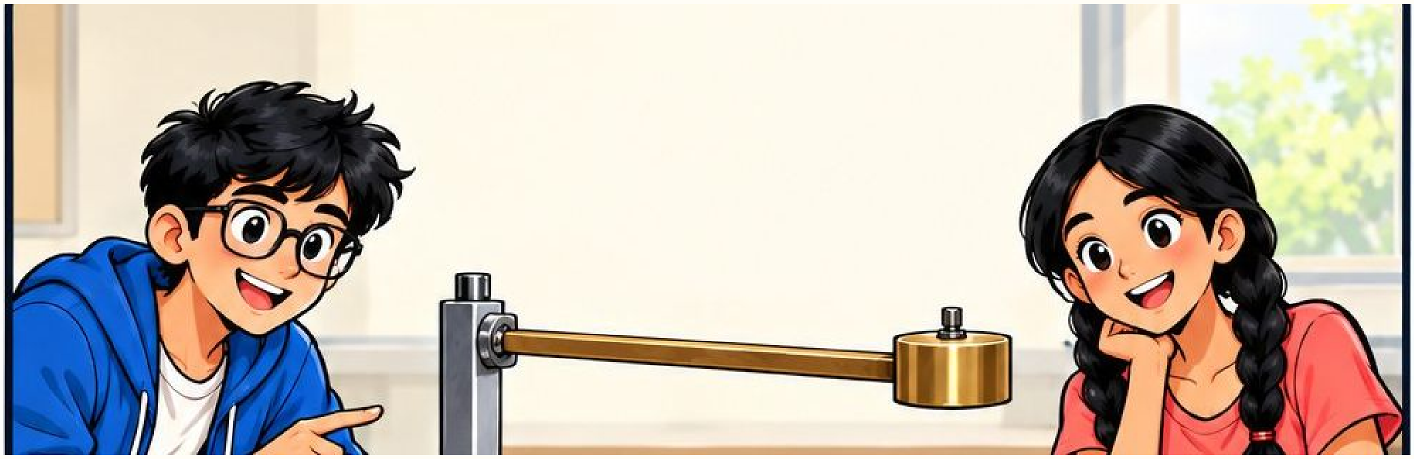
SEEMA
Solve the first-order speed equation from rest.

PROBLEM 17 • SPEED-DEPENDENT DRIVING TORQUE

A disc of moment of inertia I turns about a fixed axle under driving torque $\tau_0(1-\omega/\Omega)$ and resisting torque $c\omega$, with positive constants. Starting from rest, find terminal angular speed and the time needed to reach half that speed.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



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DRAW FIRST

On a torque-versus-ω graph, the motor line slopes downward and resisting torque slopes upward; their crossing is terminal speed.

1. $I\,d\omega/dt=\tau_0-(\tau_0/\Omega+c)\omega$. Define $K=\tau_0/\Omega+c>0$.
2. Set acceleration zero: $\omega_{term}=\tau_0/K$.
3. Solve the linear equation from rest: $\omega(t)=\omega_{term}[1-\exp(-Kt/I)]$. Half-speed occurs at $t=(I/K)\ln 2$.

EXAM PATTERN

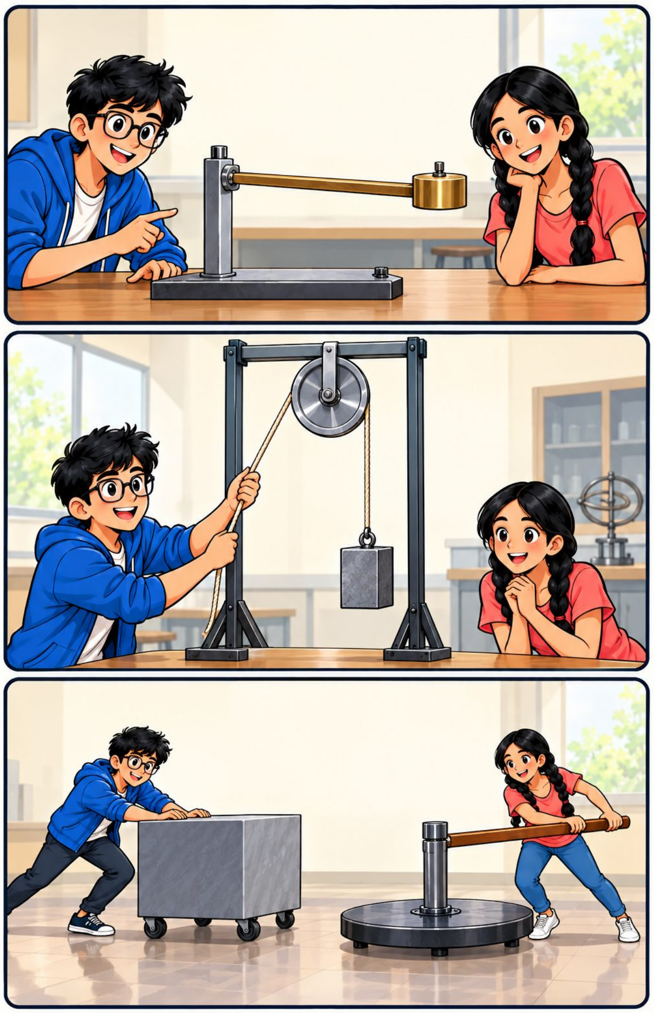
Draw each body; link ΣF to $\Sigma\tau$ with constraints.

MISCONCEPTION CHECK

A force can have zero moment about one axis.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER



ROHAN
Does the hanging mass pull with its full weight?

SEEMA
Some force accelerates it; string tension spins the cylinder.

ROHAN
Should I add cylinder translation too?

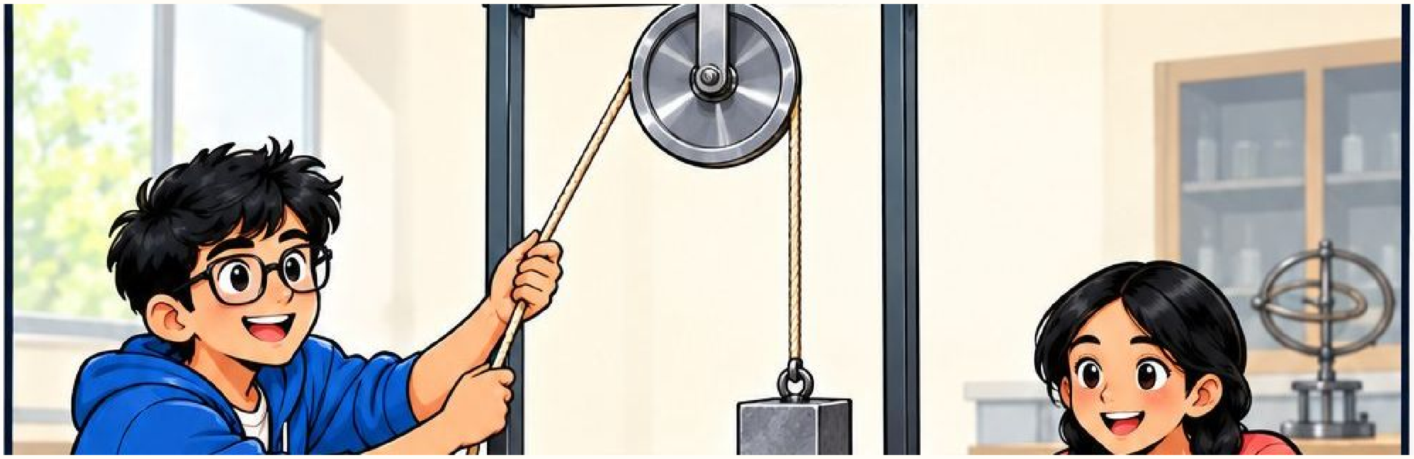
SEEMA
No. Its axle fixes the centre; use only its torque.

PROBLEM 18 • HANGING MASS AND A FIXED-AXLE CYLINDER

A light string is wound round a solid cylinder of mass M and radius R on a fixed axle. A mass m hangs from the free end. Find acceleration, tension and cylinder angular acceleration; show the limit as $M \to 0$ and specify why it differs from a cylinder moving translationally.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



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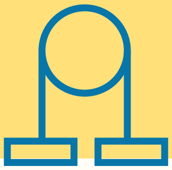


DRAW FIRST

Two separate diagrams: falling mass with mg,T; cylinder with one string torque TR. The cylinder's centre is fixed.

1. $mg - T = ma$, $TR = I\alpha$ and $a = \alpha R$.
2. With $I = MR^2/2$, $T = (M/2)a$. Thus $a = mg/(m + M/2)$, $T = mMg/[2m+M]$, $\alpha = a/R$.
3. $M \to 0$ gives $a \to g$ and $T \to 0$. Do not add a cylinder centre-of-mass acceleration term: its axle holds its centre stationary.

EXAM PATTERN

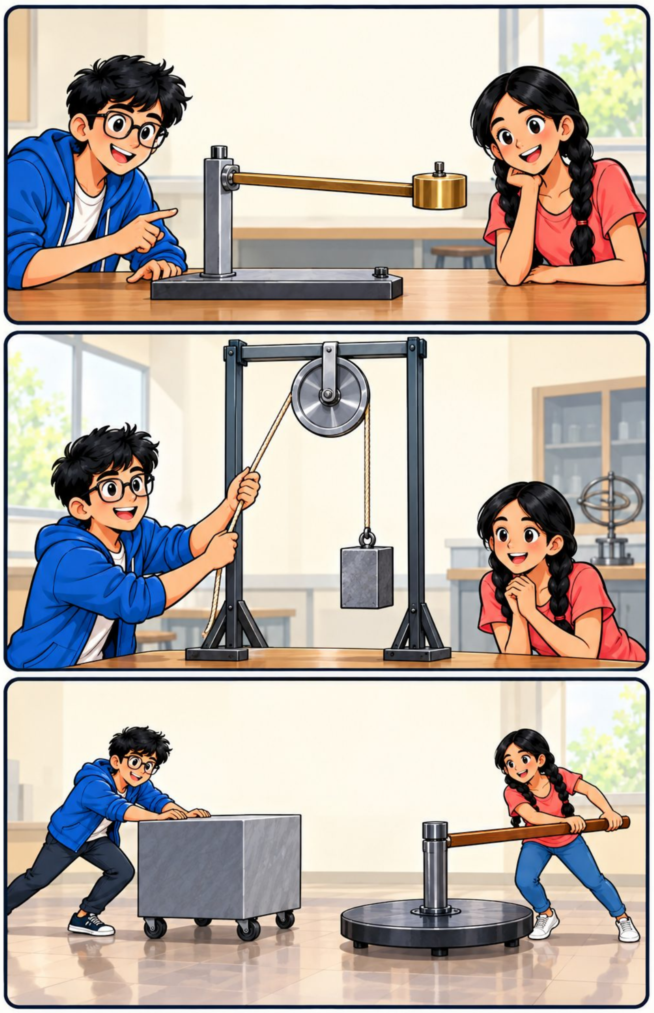
Draw each body; link ΣF to $\Sigma\tau$ with constraints.

MISCONCEPTION CHECK

A force can have zero moment about one axis.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER



ROHAN
Why can a near-end force make the rod turn opposite to it?

SEEMA
The pin is between the force and the centre.

ROHAN
So the torque sign comes from the force's position?

SEEMA
Yes. Then find the centre acceleration and pin reaction.

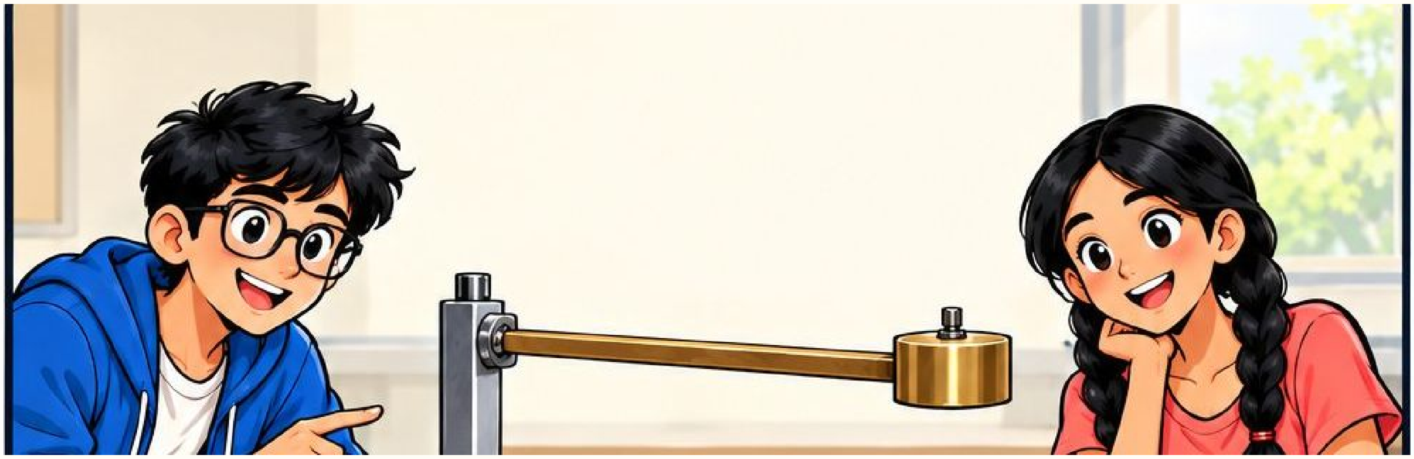
PROBLEM 19 • OFF-CENTRE PIN AND NEAR-END FORCE

A uniform rod of length L and mass M is free to rotate in a horizontal plane about a fixed vertical pin at a point L/4 from one end. A horizontal force F perpendicular to the rod is applied at its near end. Find the initial angular acceleration and the pin force immediately after release.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.

TORQUE AND ANGULAR ACCELERATION



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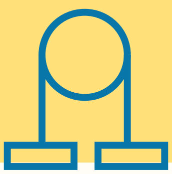


DRAW FIRST

Pin at $x=L/4$, rod CM a further L/4 to the right, near-end force F upward at $x=0$. The force turns the rod clockwise.

1. $I_{pin}=ML^2/12+M(L/4)^2=(7/48)ML^2$.

2. Signed torque $\tau=-FL/4$ gives $\alpha=-12F/(7ML)$. Initially $\omega=0$, so $a_{CM}=\alpha(L/4)=-3F/(7M)$ in the transverse direction.

3. Net transverse force is $F+H_y=Ma_{CM}$, hence $H_y=-10F/7$; $H_x=0$. The pin force can exceed the applied force in magnitude.

EXAM PATTERN

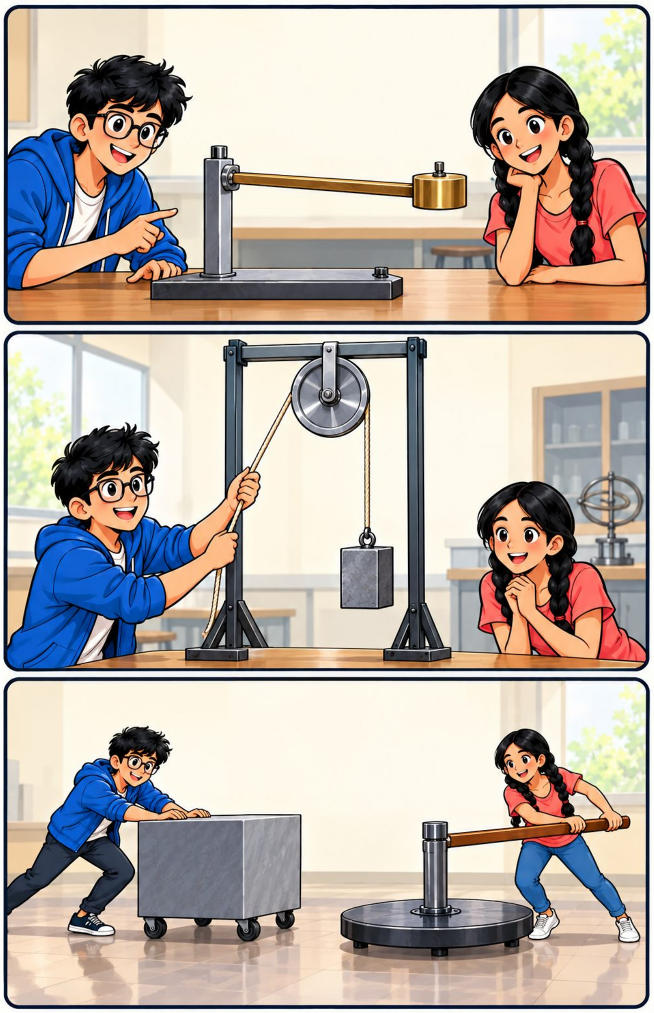
Draw each body; link ΣF to $\Sigma\tau$ with constraints.

MISCONCEPTION CHECK

A force can have zero moment about one axis.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER

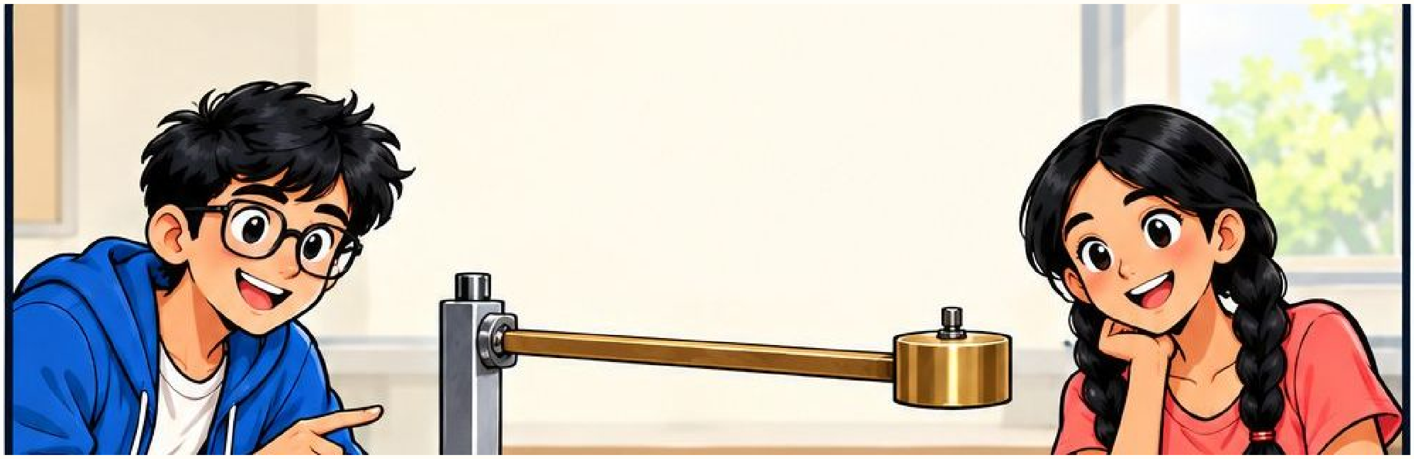


PROBLEM 20 • A BRAKE WITH TORQUE PROPORTIONAL TO ANGLE

A wheel's moment of inertia about a fixed axis is I and it starts with angular speed ω_0. A brake produces resisting torque $\tau=k\theta$, where θ measures angular displacement since braking began. Find its stopping angle and discuss whether the braking time has an elementary expression.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



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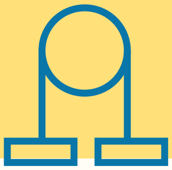


DRAW FIRST

Plot rotational kinetic energy versus θ; it is a downward parabola and reaches zero at the first stopping angle.

1. Work of the brake from 0 to θ is $-\int k\theta\, d\theta=-k\theta^2/2$.
2. At rest, $I\omega_0^2/2=k\theta_{stop}^2/2$, so $\theta_{stop}=\omega_0\sqrt{I/k}$ for $\omega_0>0$.
3. During forward motion, $\omega^2=\omega_0^2-(k/I)\theta^2$. Integrate $dt=d\theta/\omega$ from zero to θ_{stop}: $t_{stop}=(\pi/2)\sqrt{I/k}$. It **does** have an elementary expression (quarter of the associated oscillator period).

EXAM PATTERN

Draw each body; link ΣF to $\Sigma\tau$ with constraints.

MISCONCEPTION CHECK

A force can have zero moment about one axis.

Model assumptions and sign conventions are stated in the worked steps.

WORK AND ENERGY

Follow the energy through motion

Work changes mechanical energy, while friction or impact may dissipate it. A rolling rigid body has both translation and rotation energy. A stationary contact point can have zero instantaneous work even when static friction supplies torque.

$$K = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

MISCONCEPTION

Conserved energy depends on the process.

THIS CHAPTER’S PROBLEMS

21 Rod swinging through the bottom

22 Solid sphere versus spherical shell

23 Motor and opposing torque

24 Cylinder rolling uphill

25 Falling mass coupled to a spring and pulley

ROHAN & SEEMA / THINK TOGETHER

ROHAN
The rod reaches the bottom. Is its hinge force just Mg?

SEEMA
No. Find speed from the centre's drop first.

ROHAN
At the bottom the centre also accelerates upward?

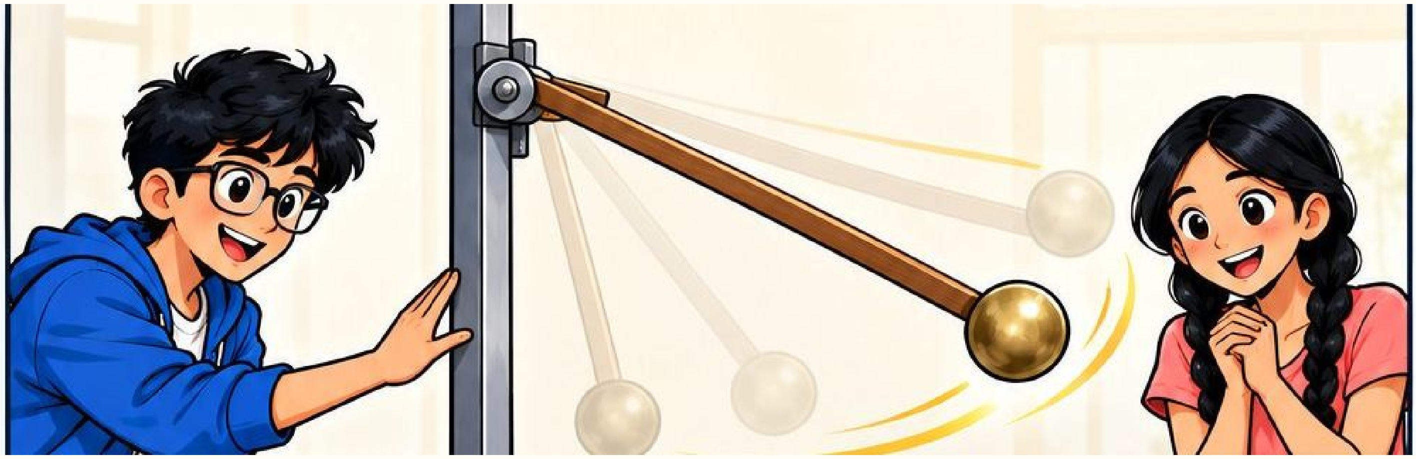
SEEMA
Exactly. Use radial acceleration in the force balance.

PROBLEM 21 • ROD SWINGING THROUGH THE BOTTOM

A uniform rod of mass M and length L is hinged at one end and starts at rest at angle θ_0 from the downward vertical. Find its speed at the lower end at the bottom and the hinge force there; assume it reaches the bottom without obstruction.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



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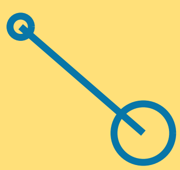


DRAW FIRST

Draw the rod initially θ_0 from downward vertical and finally vertical; the centre drops $(L/2)(1-\cos\theta_0)$.

1. Energy gives $Mg(L/2)(1-\cos\theta_0)=(1/2)(ML^2/3)\omega_b^2$; so $\omega_b^2=(3g/L)(1-\cos\theta_0)$ and lower-end speed $v_b=\sqrt{3gL(1-\cos\theta_0)}$.

2. At the bottom $\alpha=0$ because gravity's line passes through the hinge. The centre's acceleration is upward, $a_c=\omega_b^2 L/2$.

3. $H_y-Mg=M\omega_b^2 L/2$, so $H_y=Mg[1+(3/2)(1-\cos\theta_0)]$, $H_x=0$. Zero *torque* at the bottom does not mean zero hinge force.

EXAM PATTERN

Compare initial and final energy; identify nonconservative work.

MISCONCEPTION CHECK

Conserved energy depends on the process.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER

ROHAN
Which reaches the bottom first, the sphere or shell?

SEEMA
Compare how much energy each stores in spin.

ROHAN
More rotational inertia means less centre speed?

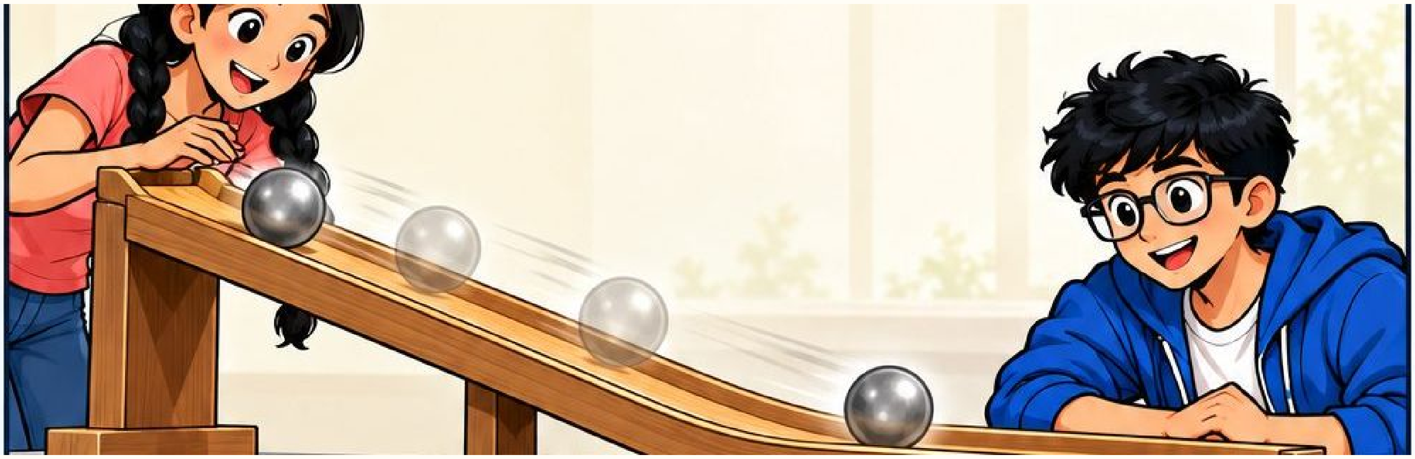
SEEMA
Yes. Check the friction each needs to roll.

PROBLEM 22 • SOLID SPHERE VERSUS SPHERICAL SHELL

A solid sphere and thin spherical shell of equal M,R roll without slipping down identical tracks with vertical drop h. Find their final speeds, ratio of descent times on a straight incline of angle θ, and friction needed at each incline angle.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



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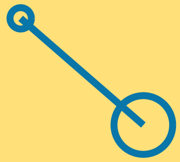


DRAW FIRST

Draw equal centre speeds but different fractions of energy in spin; label $k=I/(MR^2)$.

1. Rolling energy: $Mgh=(1/2)Mv^2(1+k)$, hence $v^2=2gh/(1+k)$.

2. Solid sphere $k=2/5$ gives $v_s^2=10gh/7$. Thin shell $k=2/3$ gives $v_h^2=6gh/5$.

3. On the same straight incline, $a=g\sin\theta/(1+k)$ and $t=\sqrt{2s/a}$. Thus $t_s/t_h=\sqrt{[(7/5)/(5/3)]}=\sqrt{21}/5$.

4. Uphill static friction magnitudes are $f_s=(2/7)Mg\sin\theta$ and $f_h=(2/5)Mg\sin\theta$. Check $\mu_s\geq f/(Mg\cos\theta)$ separately for each.

CHECK THE MODEL

Conserved energy depends on the process.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER

ROHAN
Can the rotor cross the whole angle Θ?

SEEMA
Compare motor work, braking work and initial kinetic energy.

ROHAN
Is checking only the final energy enough?

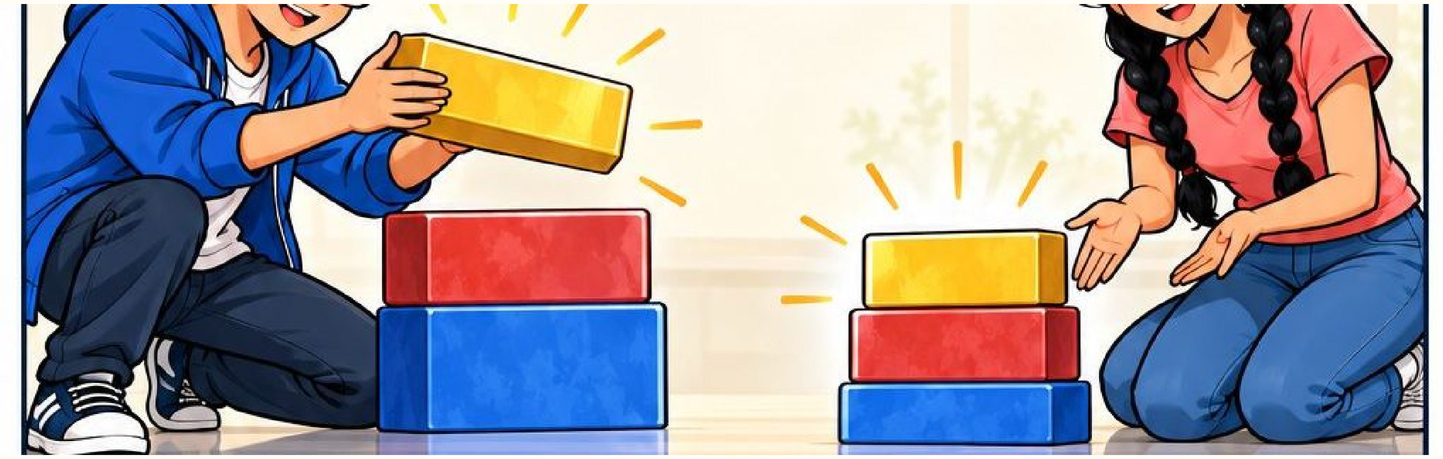
SEEMA
Here the energy curve is concave; check its endpoints.

PROBLEM 23 • MOTOR AND OPPOSING TORQUE

A motor exerts torque $\tau(\theta)=\tau_0(1-\theta/\Theta)$ on a rotor of I from $\theta=0$ to $\theta=\Theta$. A constant opposing torque τ_f also acts. Starting from ω_0, find ω at Θ and the smallest ω_0 that guarantees the rotor can traverse the entire interval.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



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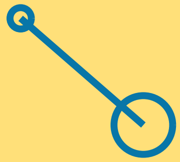


DRAW FIRST

Shade area under the motor torque-versus-angle triangle; subtract the opposing rectangle.

1. Motor work to Θ is $\tau_0\Theta/2$. Resistive work is $-\tau_f\Theta$.

2. Energy yields $\omega_\Theta^2=\omega_0^2+(\tau_0-2\tau_f)\Theta/I$; choose the nonnegative square root while turning forward.

3. $K(\theta)=I\omega_0^2/2+(\tau_0-\tau_f)\theta-\tau_0\theta^2/(2\Theta)$, a concave function. Its minimum over $0\le\theta\le\Theta$ lies at an endpoint.

4. Therefore the infimum initial speed for reaching Θ is $\sqrt{\max[0,(2\tau_f-\tau_0)\Theta/I]}$. If it arrives at Θ with zero speed, its ability to proceed beyond Θ depends on the torque law outside the stated interval.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER

ROHAN

A cylinder rolls uphill. Which way does friction act?

SEEMA

Sketch its spin and the torque needed to slow that spin.

ROHAN

So friction can point uphill too?

SEEMA

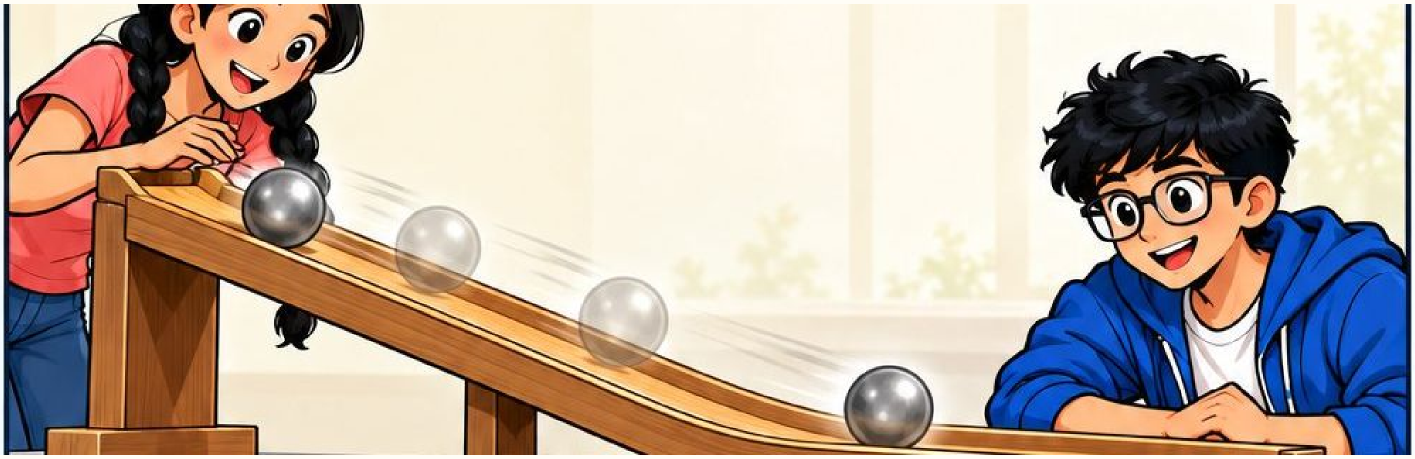
Yes. Solve translation and rotation before trusting intuition.

PROBLEM 24 • CYLINDER ROLLING UPHILL

A uniform disc of mass M and radius R rolls up a rough fixed incline with initial centre speed v_0 and no slip throughout. Find its maximum vertical rise and initial and final signs of static friction. Explain carefully whether the friction force must always be nonzero.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



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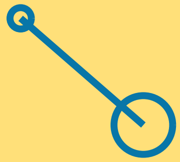


DRAW FIRST

Translation points uphill but acceleration downhill; friction points *uphill* and its torque reduces the initial clockwise spin.

1. Let uphill translation and clockwise rotation be positive. Translation: $-Mg\sin\theta+f=Ma$. Torque about centre: $-fR=I\alpha$; no-slip $a=R\alpha$.

2. With $k=I/(MR^2)=1/2$, solve $a=-g\sin\theta/(1+k)=-(2/3)g\sin\theta$ and $f=[k/(1+k)]Mg\sin\theta=(1/3)Mg\sin\theta$, uphill.

3. Initial energy is $(1/2)Mv_0^2(1+k)$, so maximum vertical rise $H=(1+k)v_0^2/(2g)=3v_0^2/(4g)$. The ideal rolling constraint gives the same uphill friction direction during ascent and immediately after reversal.

4. For $\theta>0$ and $k>0$, friction is nonzero under this model. The momentary zero speed at the turning point does not switch off the needed torque.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER

ROHAN
As the mass falls, what happens to the spring?

SEEMA
Its extension increases by the same displacement y.

ROHAN
Energy gives the turning point, but is that all?

SEEMA
Check both string tensions stay nonnegative.

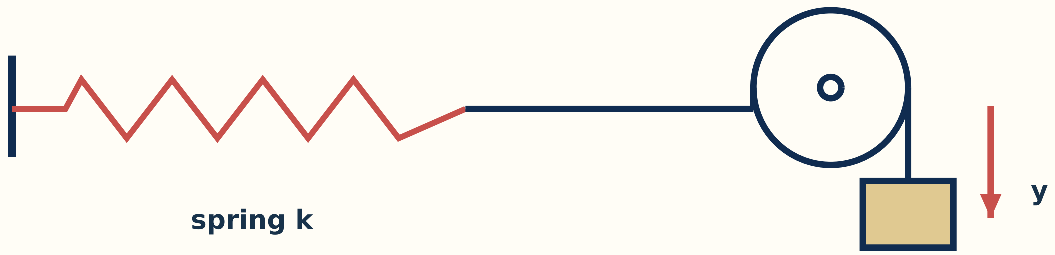
PROBLEM 25 • FALLING MASS COUPLED TO A SPRING AND PULLEY

A mass m hangs from a string wound around a massive pulley of I and radius R; the other end is attached to a horizontal spring k, arranged so a downward mass displacement y increases its extension from x_0 to x_0+y. Assume x_0≥0. With mass released at rest, formulate energy as a function of y and find the first turning point. State the valid interval before the string could slacken.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.

SPRING → MASS / SHARED STRING DISPLACEMENT



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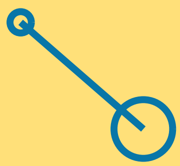
A mass m hangs from a string wound around a massive pulley of I and radius R; the other end is attached to a horizontal spring k, arranged so a downward mass displacement y increases its extension from x_0 to x_0+y. Assume $x_0 \geq 0$. With mass released at rest, formulate energy as a function of y and find the first turning point. State the valid interval before the string could slacken.



DRAW FIRST

Mark y downward for the mass and extension $x=x_0+y$. The pulley's angular speed is $\dot{y}/R$.

1. The moving kinetic energy is $K=(1/2)(m+I/R^2)\dot{y}^2$.

2. Energy relative to release is $K+(k/2)[(x_0+y)^2-x_0^2]-mgy=0$.

3. At the first nonzero turning point $K=0$: $y[kx_0+(k/2)y-mg]=0$, so $y_{turn}=2(mg-kx_0)/k$. A downward excursion requires $mg>kx_0$; otherwise the initial motion is upward or remains in equilibrium.

4. This energy result is valid only while the string remains taut and the assumed wrap/contact geometry persists. Here $a=[mg-k(x_0+y)]/(m+I/R^2)$, spring-side tension is $k(x_0+y)$, and mass-side tension is $m(g-a)$. For $x_0 \geq 0$ and the downward excursion $0 \leq y \leq y_{turn}$, both are nonnegative, so the assumed taut regime is consistent. In other configurations, check tensions anew.

Model assumptions and sign conventions are stated in the worked steps.

ROLLING WITHOUT SLIPPING

Contact creates a constraint

No slip ties a body’s contact velocity to its surface. On fixed ground $v=R\omega$; on a moving belt the bottom velocity equals belt speed. Always check whether available static friction is enough for the assumed rolling solution.

vcontact = vsurface

MISCONCEPTION

No slip is relative to the contact surface.

THIS CHAPTER’S PROBLEMS

26 Cylinder and general inertia factor

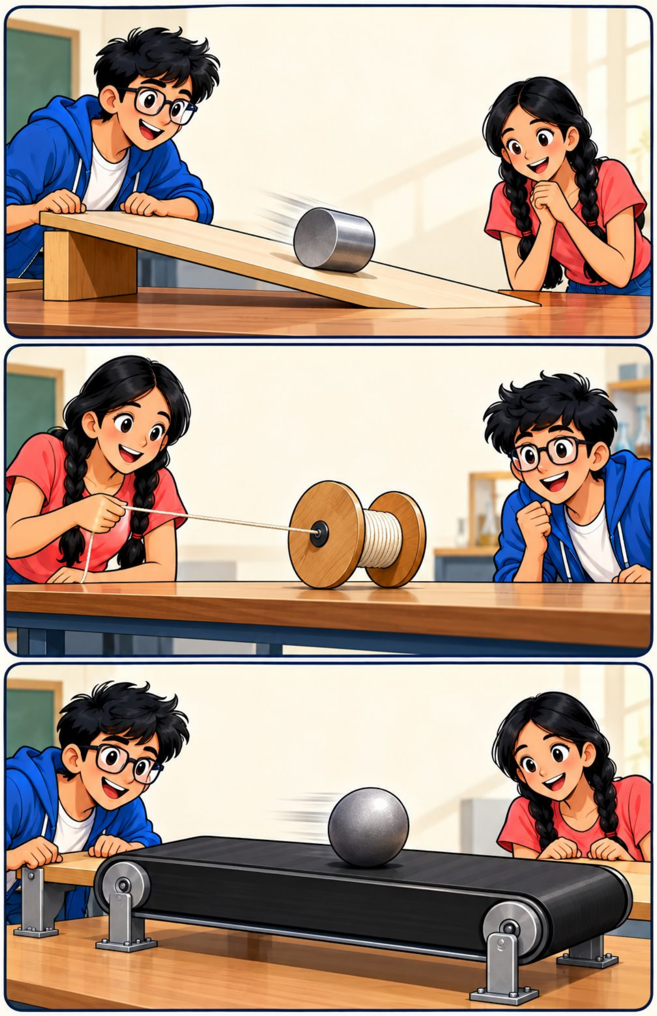
27 Spool pulled on upper or lower axle tangent

28 Sliding-to-rolling cylinder

29 Cylinder on a moving belt

30 Cylinder rolling on an accelerating plank

ROHAN & SEEMA / THINK TOGETHER



ROHAN
Why does a hollow body accelerate differently?

SEEMA
Write I as kmR^2; k measures mass distribution.

ROHAN
Does no slip automatically hold?

SEEMA
Only if available static friction meets the required value.

PROBLEM 26 • CYLINDER AND GENERAL INERTIA FACTOR

A uniform solid cylinder rolls without slipping down an incline θ. Find a, f, and the minimum coefficient μ_s. Then repeat for a body with $I=kmR^2$ and compare the $k\to 0$ and $k\to\infty$ limits, noting which idealized bodies are physically realizable.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.

PROBLEM 26 • CYLINDER AND GENERAL INERTIA FACTOR

A uniform solid cylinder rolls without slipping down an incline θ. Find a, f, and the minimum coefficient μ_s. Then repeat for a body with $I=kmR^2$ and compare the $k\to 0$ and $k\to\infty$ limits, noting which idealized bodies are physically realizable.

DRAW FIRST

On a fixed incline, draw $mg\sin\theta$ downhill; contact friction uphill generates the required clockwise spin.

1. Translation: $mg\sin\theta - f = ma$. Rotation: $fR = I\alpha$; constraint $a = \alpha R$.

2. Set $I = kmR^2$. Then $f = kma$, so $a = g\sin\theta/(1+k)$ and $f = [k/(1+k)]mg\sin\theta$.

3. Feasible static friction requires $\mu_s \geq f/(mg\cos\theta) = [k/(1+k)]\tan\theta$. For a solid cylinder $k=1/2$: $a=(2/3)g\sin\theta$, $f=(1/3)mg\sin\theta$ and $\mu_s \geq (1/3)\tan\theta$.

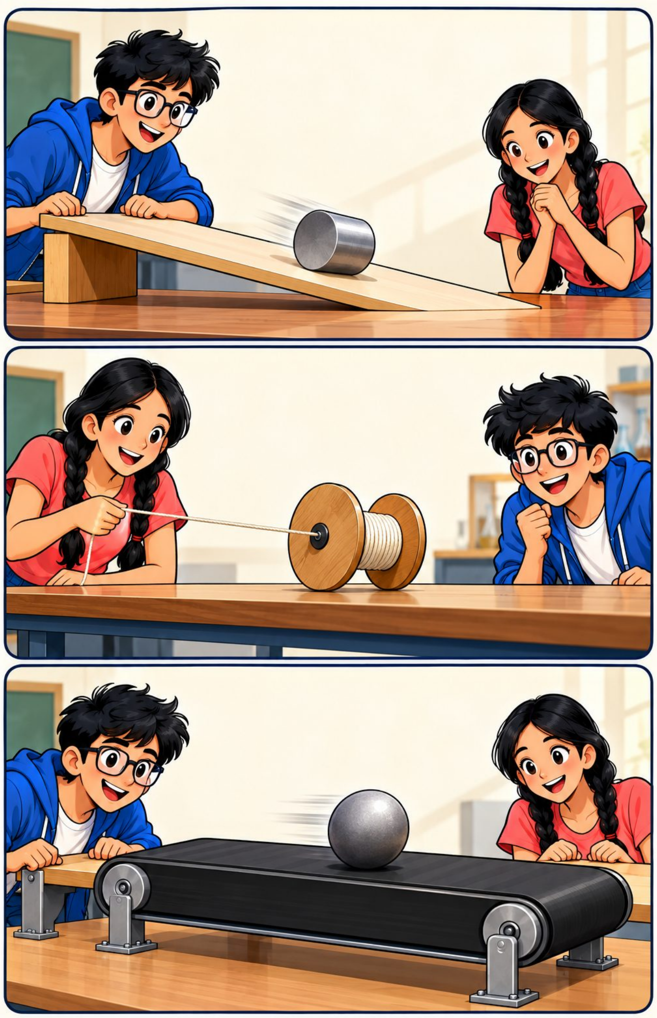
4. $k\to 0$ gives pointlike/no-spin-inertia behavior $a\to g\sin\theta$. Algebraically $k\to\infty$ gives $a\to 0$ and $f\to mg\sin\theta$, but ordinary bodies confined within a radius R about their centre have $k\leq 1$; the large-k limit needs an altered mass model.

CHECK THE MODEL

No slip is relative to the contact surface.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER



ROHAN
Can pulling a spool to the right make it roll left?

SEEMA
Take torque about the floor contact for each string tangent.

ROHAN
The lower string has a critical angle?

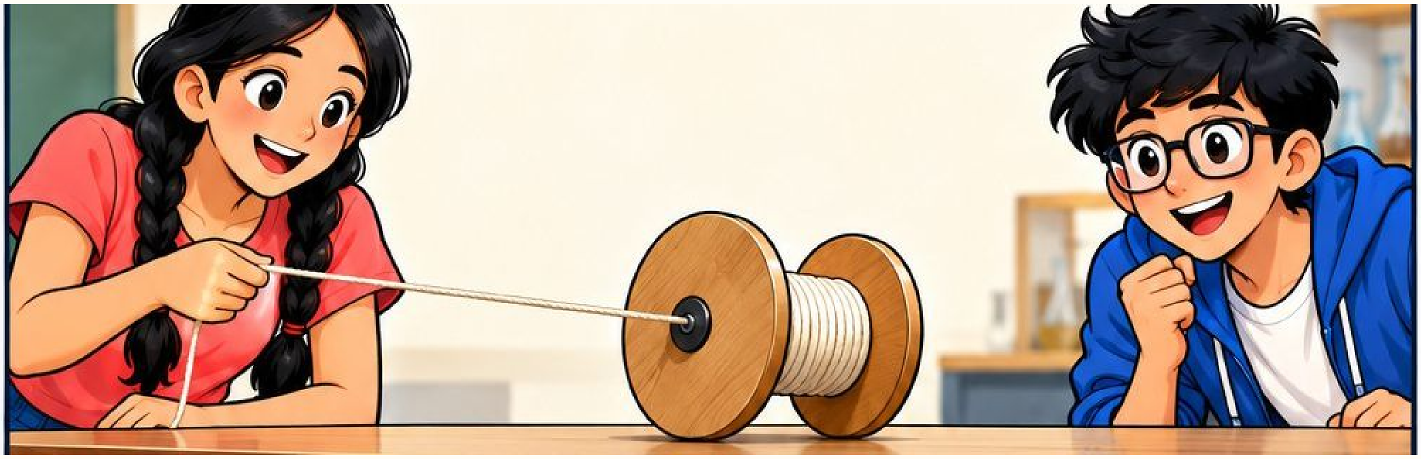
SEEMA
Yes. Compare R cosθ with the axle radius r.

PROBLEM 27 • SPOOL PULLED ON UPPER OR LOWER AXLE TANGENT

A spool has outer radius R, axle radius $r<R$, mass M and I about its centre. A string tangent to the inner axle is pulled at angle θ above the horizontal while the outer rim rolls without slipping on the floor. Determine the direction of centre motion as a function of θ for both string-winding geometries; identify the critical angle(s).

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



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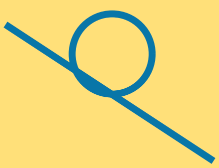


DRAW FIRST

Use the floor contact as an instantaneous torque axis. Upper tangent force has a lever arm $R\cos\theta+r$; lower tangent has $R\cos\theta-r$.

1. Take rightward centre acceleration positive and clockwise angular acceleration $\alpha=a/R$. Around the floor contact, effective fixed-contact inertia for rolling is $I+MR^2$.

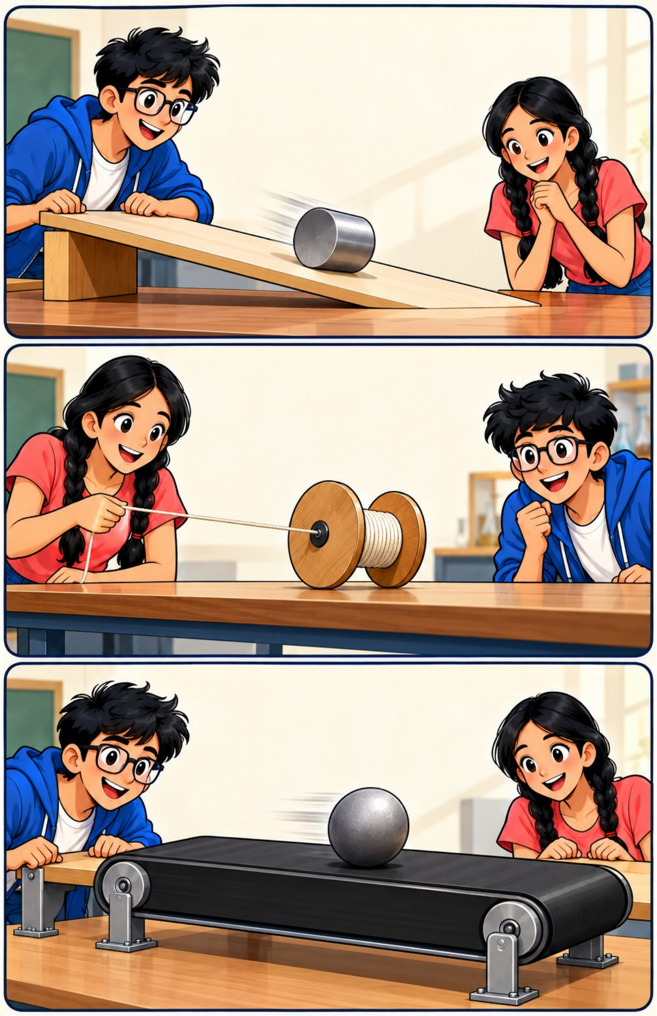
2. The force supplies clockwise driving moment $F(R\cos\theta+r)$ for the upper tangent: $a_{upper}=FR(R\cos\theta+r)/(I+MR^2)$, always rightward for $0\le\theta<\pi/2$.

3. For the lower tangent, $a_{lower}=FR(R\cos\theta-r)/(I+MR^2)$. It moves right for $\cos\theta>r/R$, left for $\cos\theta<r/R$, and has zero initial acceleration at $\theta_c=\arccos(r/R)$.

4. Verify with $\Sigma F_x=F\cos\theta+f=Ma$ and torque about the centre. The sign result assumes the string exits the specified upper/lower side and sufficient static friction.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER



ROHAN
The cylinder slides and spins. When will it roll cleanly?

SEEMA
Track the bottom slip speed $u=v-R\omega$.

ROHAN
Does friction always point left?

SEEMA
It opposes the sign of u, which can be either way.

PROBLEM 28 • SLIDING-TO-ROLLING CYLINDER

A cylinder initially has centre speed v_0 to the right and angular speed ω_0 clockwise on a rough horizontal floor with kinetic friction coefficient μ. Find the time and centre displacement until pure rolling begins for all three regimes $v_0-R\omega_0$ positive, zero, and negative, with signed friction explicitly shown.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.

ROLLING WITHOUT SLIPPING

PROBLEM 28 • SLIDING-TO-ROLLING CYLINDER

A cylinder initially has centre speed v_0 to the right and angular speed ω_0 clockwise on a rough horizontal floor with kinetic friction coefficient μ. Find the time and centre displacement until pure rolling begins for all three regimes $v_0 - R\omega_0$ positive, zero, and negative, with signed friction explicitly shown.

DRAW FIRST

At the bottom, draw relative slip $u = v - R\omega$ with clockwise ω positive. Friction points opposite the sign of u.

1. For a solid cylinder, $a = -\mu g \operatorname{sgn}(u)$, $\alpha_{clockwise} = (2\mu g/R)\operatorname{sgn}(u)$, so $\dot{u} = a - R\alpha = -3\mu g \operatorname{sgn}(u)$.

2. If $u_0 = v_0 - R\omega_0 \neq 0$, rolling begins at $t_r = |u_0|/(3\mu g)$. The centre displacement is $x_r = v_0 t_r - (\mu g/2)\operatorname{sgn}(u_0) t_r^2$.

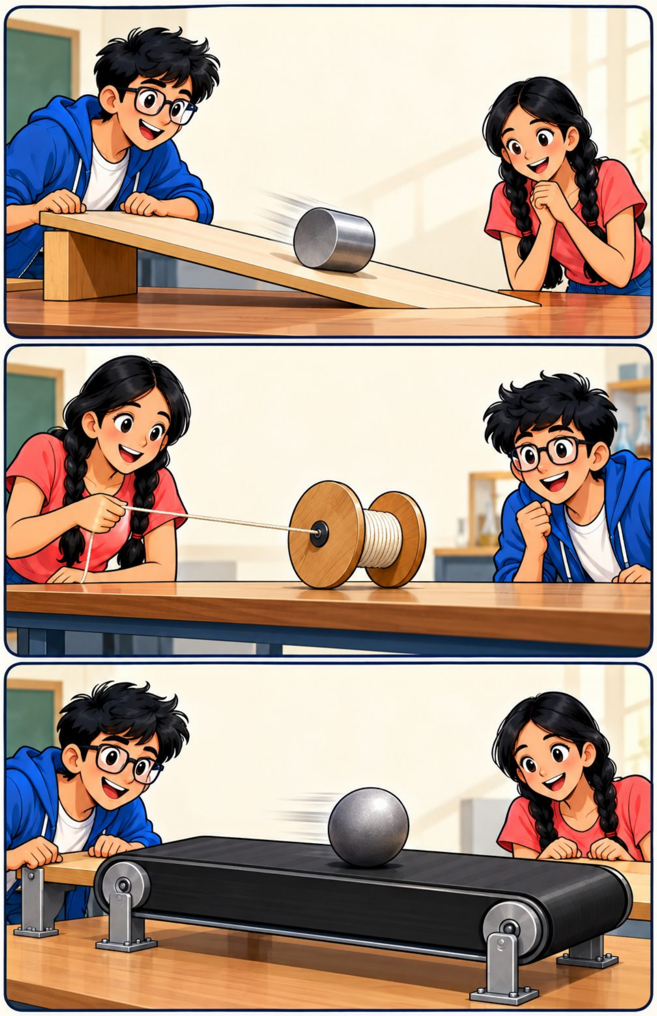
3. At that instant $v_r = v_0 - u_0/3 = (2v_0 + R\omega_0)/3$ and $\omega_r = v_r/R$. If $u_0 = 0$, $t_r = 0$ and kinetic friction never acts in this ideal setup.

4. If the computed x_r is negative, it denotes displacement left of the starting point; do not silently replace it by a path length.

Model assumptions and sign conventions are stated in the worked steps.

ROLLING WITHOUT SLIPPING

ROHAN & SEEMA / THINK TOGETHER



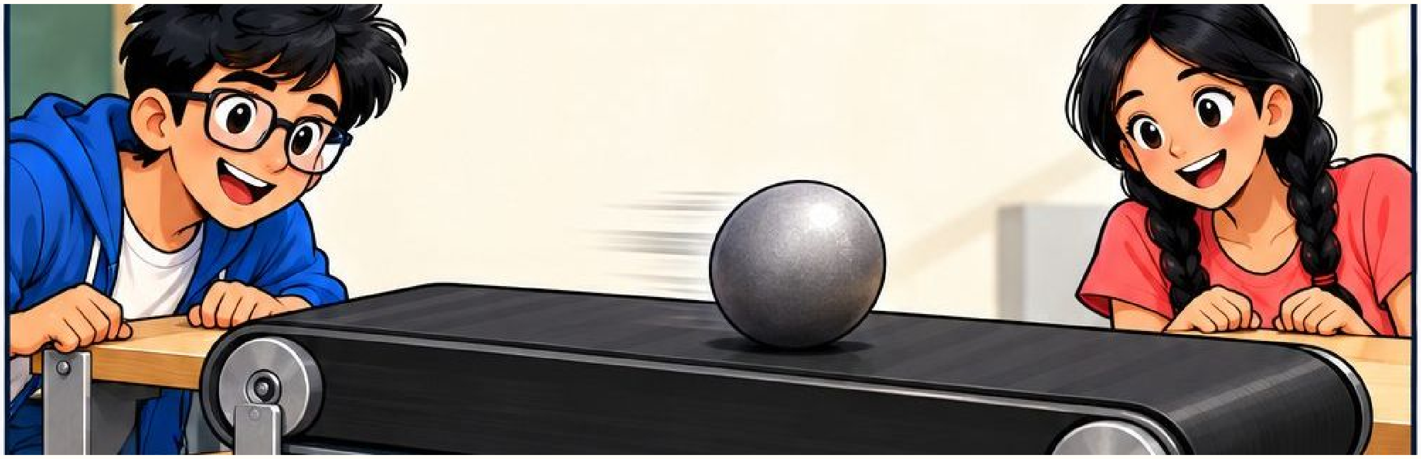
PROBLEM 29 • CYLINDER ON A MOVING BELT

A solid sphere rests on a conveyor belt moving right at speed U. At t=0 it has zero spin and centre speed v_0. With kinetic friction μ until no slip, find the final centre speed, final spin, and the time to reach rolling relative to the belt. Treat belt speed as externally maintained.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.

ROLLING WITHOUT SLIPPING



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A solid sphere rests on a conveyor belt moving right at speed U. At t=0 it has zero spin and centre speed v_0. With kinetic friction μ until no slip, find the final centre speed, final spin, and the time to reach rolling relative to the belt. Treat belt speed as externally maintained.

DRAW FIRST

Compare the bottom surface velocity $v-R\omega$ with belt speed U, not with zero.

1. Initial belt-relative slip $u_0=v_0-U$. While sliding, $\dot{u}=-3\mu g\ \mathrm{sgn}(u)$.

2. Therefore $t_r=|v_0-U|/(3\mu g)$, $v_f=v_0-u_0/3=(2v_0+U)/3$, and signed clockwise $\omega_f=2u_0/(3R)$.

3. Check $v_f-R\omega_f=U$. The belt's external drive can add or remove mechanical energy; do not impose cylinder energy conservation.

EXAM PATTERN

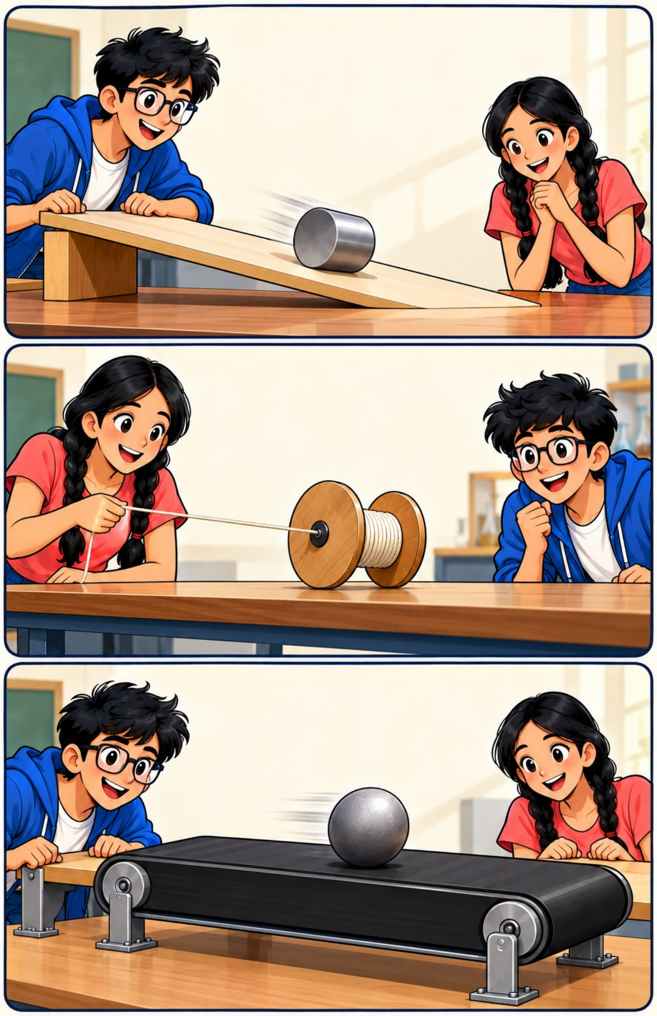
Contact velocity first; then translation and torque.

MISCONCEPTION CHECK

No slip is relative to the contact surface.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER



ROHAN
The plank also moves. What is the no-slip condition?

SEEMA
Match the cylinder contact acceleration to the plank's.

ROHAN
So friction acts oppositely on the two bodies?

SEEMA
Exactly. Draw separate diagrams before solving.

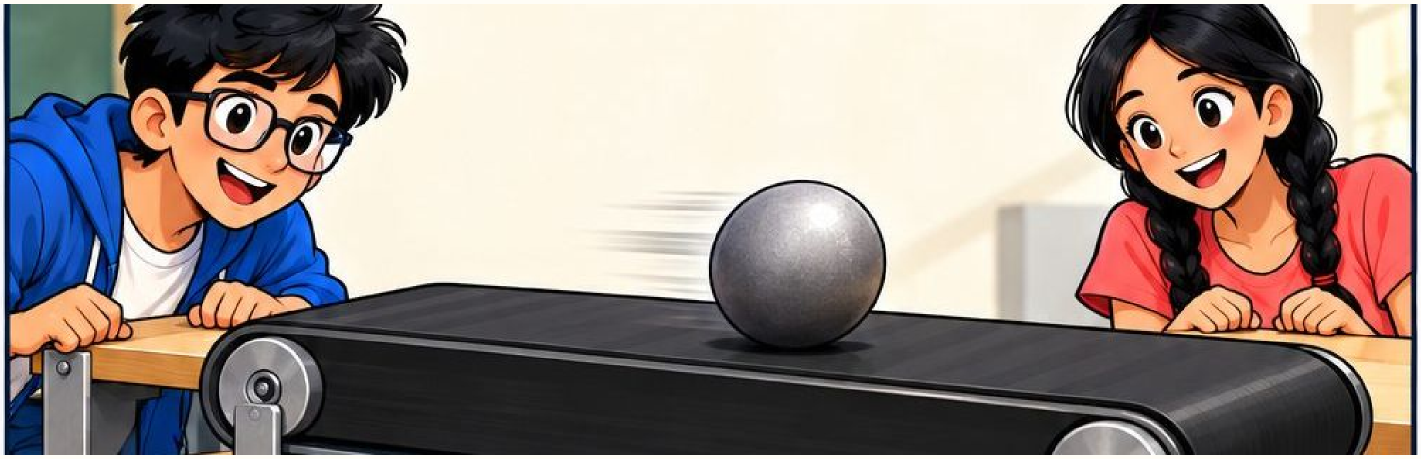
PROBLEM 30 • CYLINDER ROLLING ON AN ACCELERATING PLANK

A solid cylinder rolls without slipping on a horizontal plank of mass P which slides without friction on the ground. A horizontal force F is applied to the cylinder's centre. Find both translational accelerations, angular acceleration and contact friction, assuming sufficient static friction.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.

ROLLING WITHOUT SLIPPING



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A solid cylinder rolls without slipping on a horizontal plank of mass P which slides without friction on the ground. A horizontal force F is applied to the cylinder's centre. Find both translational accelerations, angular acceleration and contact friction, assuming sufficient static friction.

DRAW FIRST

Separate cylinder and plank. Let rightward friction on the cylinder be signed f; the plank receives $-f$. No-slip applies to *relative* contact acceleration.

1. Cylinder: $Ma_c=F+f$; clockwise angular acceleration $\alpha=-fR/I$. Plank: $Pa_p=-f$.

2. No-slip at the interface: $a_c-R\alpha=a_p$. Substitute to find $f=-F/[1+MR^2/I+M/P]$.

3. For a solid cylinder $I=MR^2/2$: $f=-FP/(3P+M)$, $a_p=F/(3P+M)$, $a_c=F(2P+M)/[M(3P+M)]$, and $\alpha=2FP/[MR(3P+M)]$ clockwise.

4. Friction acts left on the cylinder, right on the plank. Require $|f|\le\mu_s N$ for the assumed no-slip solution.

CHECK THE MODEL

No slip is relative to the contact surface.

Model assumptions and sign conventions are stated in the worked steps.

ANGULAR MOMENTUM AND COLLISIONS

Choose the axis during impact

Angular momentum depends on an axis. In a brief pinned collision, the pivot impulse has zero moment about the pivot, so angular momentum there may survive even while kinetic energy is lost. Check the external torque window.

$\tau_{ext} = dL/dt$

MISCONCEPTION

Angular momentum is conserved about a chosen axis only.

THIS CHAPTER’S PROBLEMS

31 Particle sticks to a pinned rod

32 Skater pulls masses inward

33 Mass sticks gently to a rotating disc

34 Central force and circular orbit

35 Simultaneous impacts on a free rod

ROHAN & SEEMA / THINK TOGETHER

ROHAN
The particle sticks. Can I conserve energy at impact?

SEEMA
Use angular momentum about the pin during the short collision.

ROHAN
Only the velocity across the rod contributes?

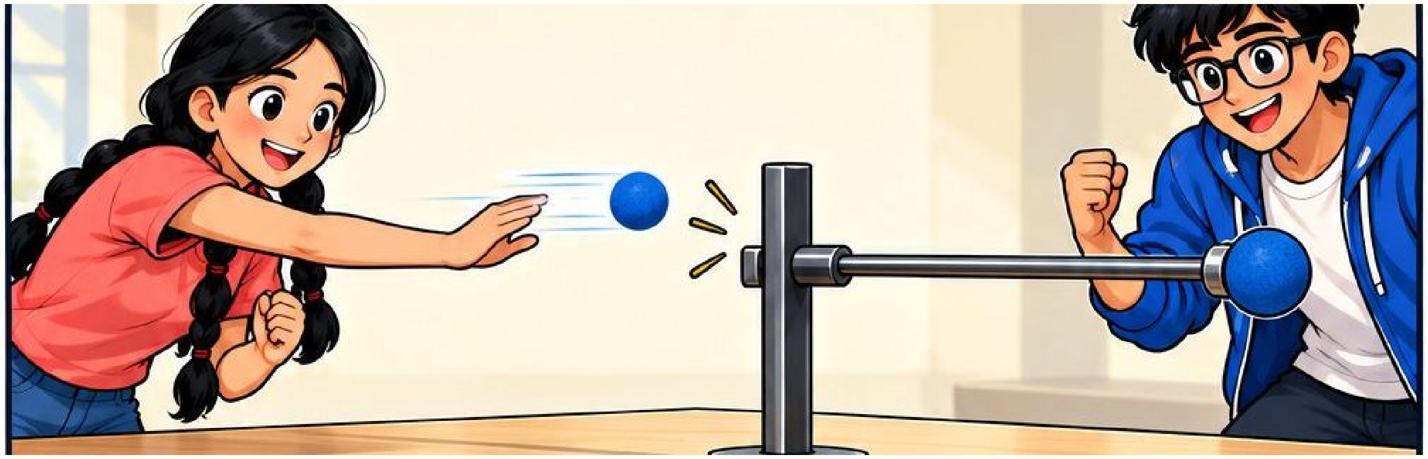
SEEMA
Yes. Compute the energy loss afterward.

PROBLEM 31 • PARTICLE STICKS TO A PINNED ROD

A particle of mass m moving at speed v hits and sticks to the end of a uniform rod of length L and mass M, pivoted at its other end. Its incident velocity makes angle φ with the rod. Find the angular speed just after impact and kinetic energy lost. Explain why angular momentum about the pivot can be conserved during impact even though linear momentum is not.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



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A particle of mass m moving at speed v hits and sticks to the end of a uniform rod of length L and mass M, pivoted at its other end. Its incident velocity makes angle φ with the rod. Find the angular speed just after impact and kinetic energy lost. Explain why angular momentum about the pivot can be conserved during impact even though linear momentum is not.

DRAW FIRST

Draw the incident velocity component perpendicular to the rod; the component along the rod contributes no angular momentum about the pin.

1 During short impact, the pin impulse has zero moment about its own location. Initial $L_{pivot}=mvL\sin\varphi$, with sign set by the approach direction.

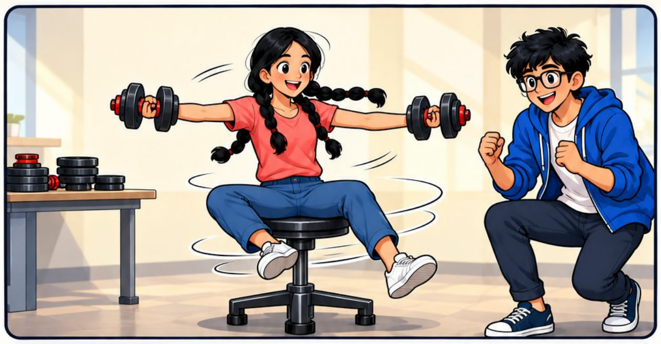
2 Final inertia is $I=(M/3+m)L^2$. Hence $\omega=mvL\sin\varphi/[(M/3+m)L^2]=mv\sin\varphi/[L(M/3+m)]$.

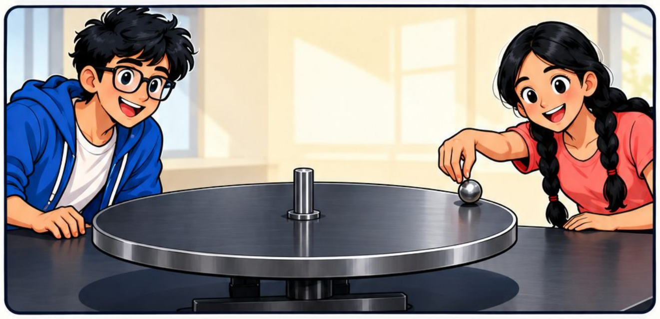
3 The loss from the particle's initial kinetic energy is $\Delta E=(1/2)mv^2-(1/2)I\omega^2=(1/2)mv^2-m^2v^2\sin^2\varphi/[2(M/3+m)]$. The axial motion and inelastic sticking both contribute to loss.

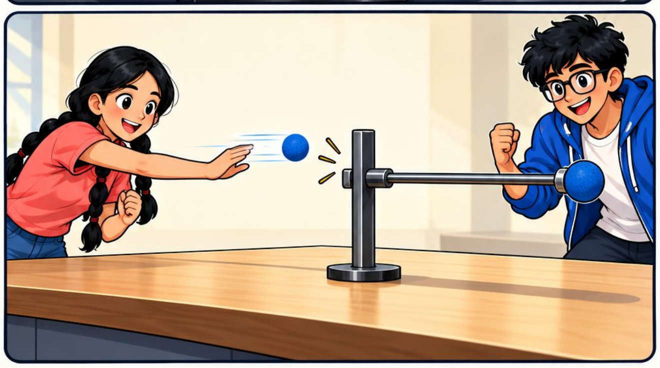
4 Linear momentum need not be conserved because the pin supplies an external impulse.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER







ROHAN
The skater pulls weights inward. Why does spin speed up?

SEEMA
External torque is negligible, so angular momentum remains.

ROHAN
Where does the extra kinetic energy come from?

SEEMA
From work done pulling the masses inward.

PROBLEM 32 • SKATER PULLS MASSES INWARD

A skater modeled as two point masses m at radius r and a torso of inertia I_0 rotates at ω_0. The masses are pulled to r/2. Find final ω, work done by the skater, and the ratio of angular kinetic energies.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.

ANGULAR MOMENTUM AND COLLISIONS

PROBLEM 32 • SKATER PULLS MASSES INWARD

A skater modeled as two point masses m at radius r and a torso of inertia I_0 rotates at ω_0. The masses are pulled to r/2. Find final ω, work done by the skater, and the ratio of angular kinetic energies.

DRAW FIRST

Draw two masses at r and then r/2; external torque about the axle is zero, while the skater does internal work.

1. $I_i = I_0 + 2mr^2$ and $I_f = I_0 + 2m(r/2)^2 = I_0 + mr^2/2$.

2. Angular momentum conservation gives $\omega_f = (I_i/I_f)\omega_0$.

3. $K_f/K_i = I_i/I_f$; work done by the skater is $W = K_f - K_i = (1/2)I_i\omega_0^2(I_i/I_f - 1) > 0$. Conserved angular momentum does **not** mean conserved kinetic energy.

EXAM PATTERN

During impact choose an axis with negligible external impulse torque.

MISCONCEPTION CHECK

Angular momentum is conserved about a chosen axis only.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER

ROHAN
A mass lands on the spinning disc. Which law survives?

SEEMA
Use angular momentum about the fixed vertical axle.

ROHAN
Does the larger final I mean slower rotation?

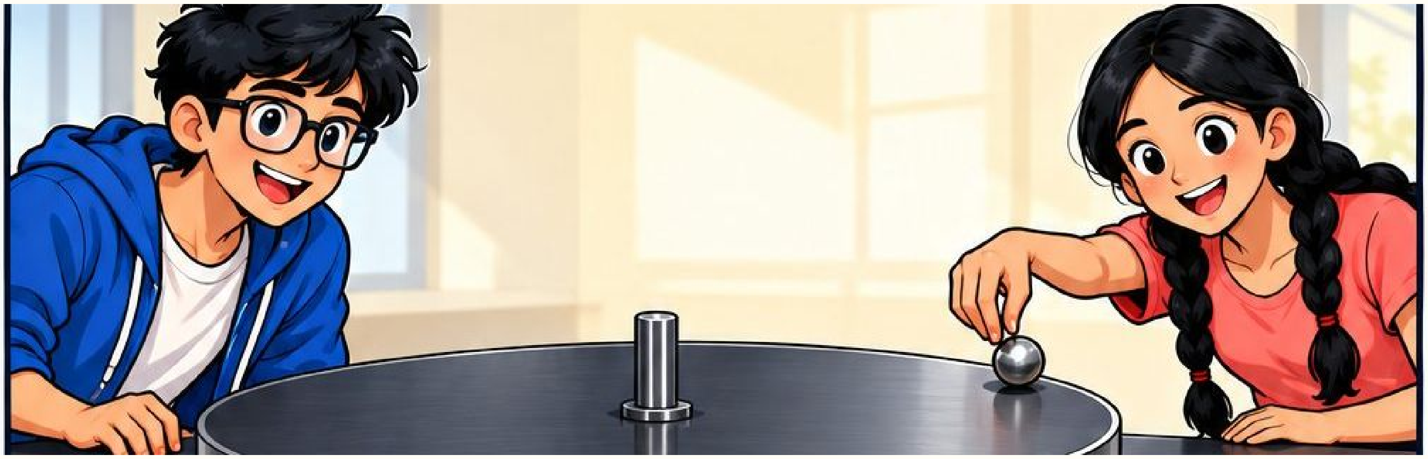
SEEMA
Yes. Then subtract energies to find the loss.

PROBLEM 33 • MASS STICKS GENTLY TO A ROTATING DISC

A uniform disc rotates at ω_0 about a frictionless vertical axle. A small mass m is lowered gently with negligible vertical impact speed and sticks at radius a. Find the final angular velocity and mechanical energy lost; specify the axis and external torques considered.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



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A uniform disc rotates at ω_0 about a frictionless vertical axle. A small mass m is lowered gently with negligible vertical impact speed and sticks at radius a. Find the final angular velocity and mechanical energy lost; specify the axis and external torques considered.

DRAW FIRST

The small mass arrives with negligible angular momentum about the vertical axle; once attached it enlarges I.

1. Initial $L=I\omega_0$, final $I'=I+ma^2$. Thus $\omega_f=I\omega_0/(I+ma^2)$.

2. The rotational kinetic energy lost is $(1/2)I\omega_0^2-(1/2)(I+ma^2)\omega_f^2=[Ima^2/(2(I+ma^2))]\omega_0^2$.

3. The question's gentle placement avoids unspecified vertical impact energy. The axle may apply a force but negligible torque about its axis.

EXAM PATTERN

During impact choose an axis with negligible external impulse torque.

MISCONCEPTION CHECK

Angular momentum is conserved about a chosen axis only.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER

ROHAN
Can a strange central force hold a circular orbit?

SEEMA
Build the effective potential at fixed angular momentum.

ROHAN
How do I tell if the orbit is stable?

SEEMA
Find a stationary radius and test the second derivative.

PROBLEM 34 • CENTRAL FORCE AND CIRCULAR ORBIT

A particle moves under an attractive central force $F(r)=-kr^2$ along the radial direction. State what is conserved, derive its effective potential for nonzero angular momentum L, and determine whether a circular orbit is stable.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.

PROBLEM 34 • CENTRAL FORCE AND CIRCULAR ORBIT

A particle moves under an attractive central force $F(r)=-kr^2$ along the radial direction. State what is conserved, derive its effective potential for nonzero angular momentum L, and determine whether a circular orbit is stable.

DRAW FIRST

Plot effective potential: centrifugal term rises at small r and $kr^3/3$ rises at large r, making a well.

1. A central force exerts zero torque about its centre, so $L=mr^2\dot{\varphi}$ is conserved. Since $F_r=-kr^2$, potential $U(r)=kr^3/3$ (up to a constant).

2. $U_{eff}(r)=L^2/(2mr^2)+kr^3/3$. A circular orbit has $U_{eff}'=-L^2/(mr^3)+kr^2=0$, so $r_c=[L^2/(mk)]^{(1/5)}$.

3. $U_{eff}''=3L^2/(mr^4)+2kr>0$ at r_c: the circular orbit is radially stable for small perturbations.

EXAM PATTERN

During impact choose an axis with negligible external impulse torque.

MISCONCEPTION CHECK

Angular momentum is conserved about a chosen axis only.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER

ROHAN
The two projectiles have opposite velocities. Does the rod move?

SEEMA
Their linear momenta cancel; their torques add.

ROHAN
What if both velocities point the same way?

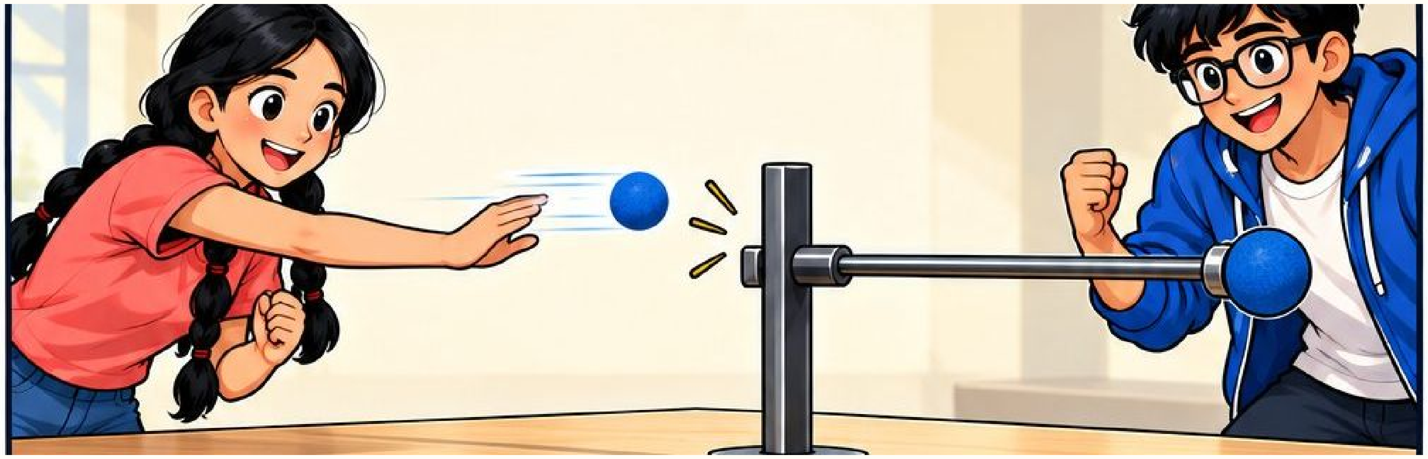
SEEMA
Then moment cancels and the joined body translates.

PROBLEM 35 • SIMULTANEOUS IMPACTS ON A FREE ROD

A horizontal uniform rod of mass M and length L is free to rotate about its centre. Two equal masses m strike and stick to its ends simultaneously with velocities v and −v perpendicular to the rod. Find final angular speed and energy loss. Repeat conceptually if both velocities are instead v in the same direction.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



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A horizontal uniform rod of mass M and length L is free to rotate about its centre. Two equal masses m strike and stick to its ends simultaneously with velocities v and −v perpendicular to the rod. Find final angular speed and energy loss. Repeat conceptually if both velocities are instead v in the same direction.

DRAW FIRST

Put the rod ends at $x=\pm L/2$. For opposite transverse velocities choose left $+v\hat{y}$ and right $-v\hat{y}$; their linear momenta cancel but torques add.

1. Opposite velocities give initial $L_z=-mvL$ about the rod's centre. Final inertia $I=ML^2/12+2m(L/2)^2=L^2(M/12+m/2)$, so $\omega_f=-mvL/I$. The centre does not translate because total linear momentum is zero.

2. Initial energy is mv^2. Final rotational energy is $L_z^2/(2I)$; heat/deformation loss is $mv^2-m^2v^2L^2/(2I)$.

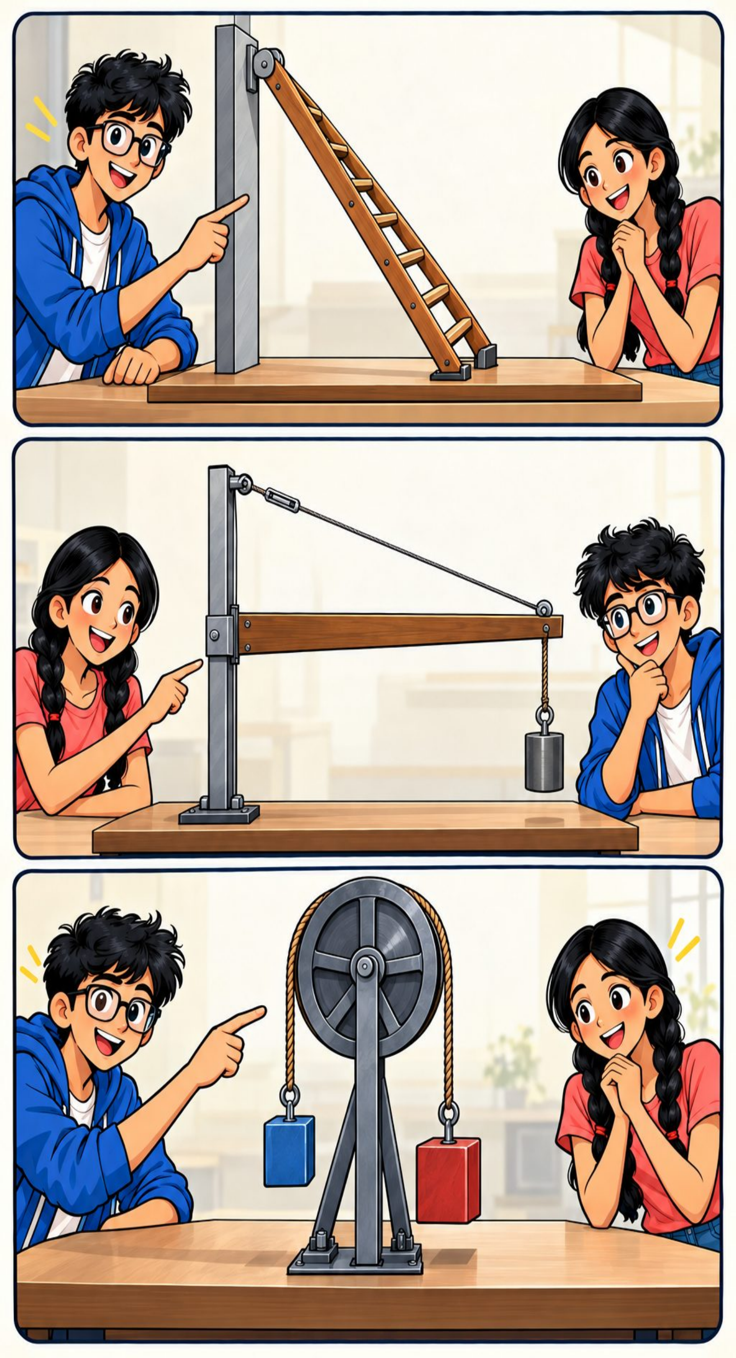
3. If both incident velocities are $+v\hat{y}$, initial L_z about the centre is zero and total momentum is $2mv\hat{y}$. The joined system translates at $V=2mv/(M+2m)$ and has $\omega_f=0$; kinetic-energy loss is $mv^2-(1/2)(M+2m)V^2$.

CHECK THE MODEL

Angular momentum is conserved about a chosen axis only.

Model assumptions and sign conventions are stated in the worked steps.

EQUILIBRIUM AND COUPLED BODIES



Tensions and supports are distinct

A massive pulley needs two distinct string tensions to turn. For beams and ladders, pick a pivot that removes unknown reactions, then use force balance as well. The final tensions and friction must satisfy physical constraints.

$\Sigma F = ma$; $\Sigma \tau = I\alpha$

MISCONCEPTION

A massive pulley has unequal tensions.

THIS CHAPTER'S PROBLEMS

36 Atwood machine with a massive pulley

37 Ladder, wall, ground and person

38 Cable-supported loaded beam

39 Block, rough table and hanging mass

40 Beam between two frictionless surfaces

ROHAN & SEEMA / THINK TOGETHER

ROHAN
Can we call both pulley tensions T?

SEEMA
A massive pulley needs their difference to accelerate.

ROHAN
So I write two block equations and one torque equation?

SEEMA
Yes. Tie all accelerations with no slip.

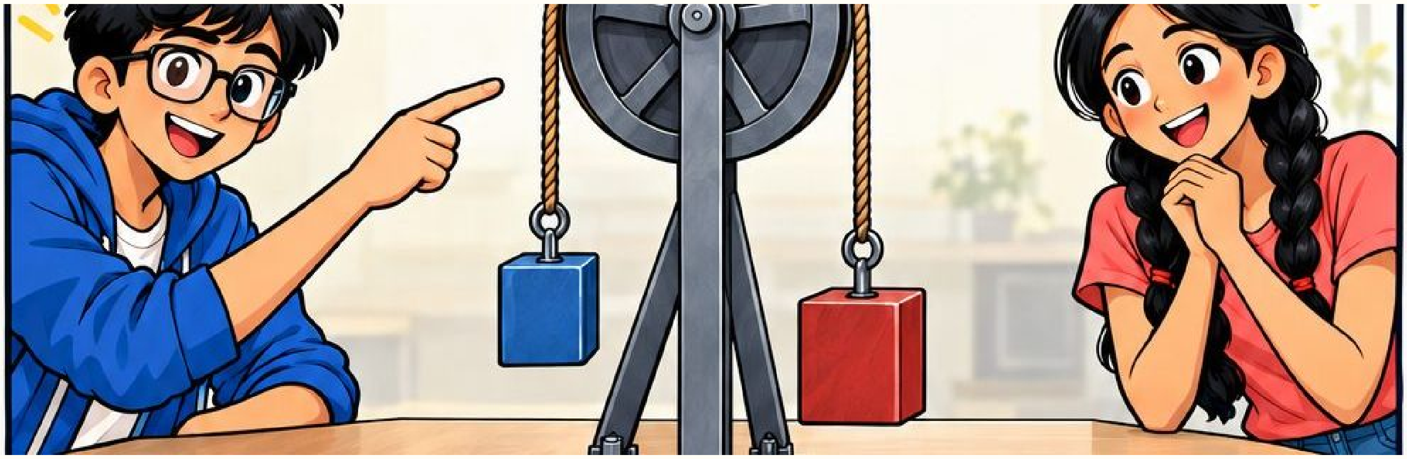
PROBLEM 36 • ATWOOD MACHINE WITH A MASSIVE PULLEY

Masses m_1 and m_2 hang on opposite sides of a pulley with mass M, radius R and $I=MR^2/2$. String is light, inextensible and does not slip. Derive a, both tensions and pulley angular acceleration for $m_2>m_1$; determine the limit $M\to0$.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.

EQUILIBRIUM AND COUPLED BODIES



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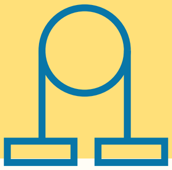


DRAW FIRST

Draw T_1 and T_2 as separate arrows. Their difference, not either one alone, turns the pulley.

1. Take m_2 downward: $m_2g-T_2=m_2a$, $T_1-m_1g=m_1a$.

2. Pulley: $(T_2-T_1)R=I(a/R)=(M/2)Ra$. Add equations: $a=(m_2-m_1)g/(m_1+m_2+M/2)$.

3. $T_1=m_1(g+a)$, $T_2=m_2(g-a)$, $\alpha=a/R$. As $M\to 0$, $T_2-T_1\to 0$ and the usual massless-pulley formula follows.

EXAM PATTERN

Separate support forces and cable tensions.

MISCONCEPTION CHECK

A massive pulley has unequal tensions.

Model assumptions and sign conventions are stated in the worked steps.

EQUILIBRIUM AND COUPLED BODIES

ROHAN & SEEMA / THINK TOGETHER

ROHAN
How far up the ladder can a person climb?

SEEMA
Take torque about its foot, then check ground friction.

ROHAN
The wall contributes only a horizontal force?

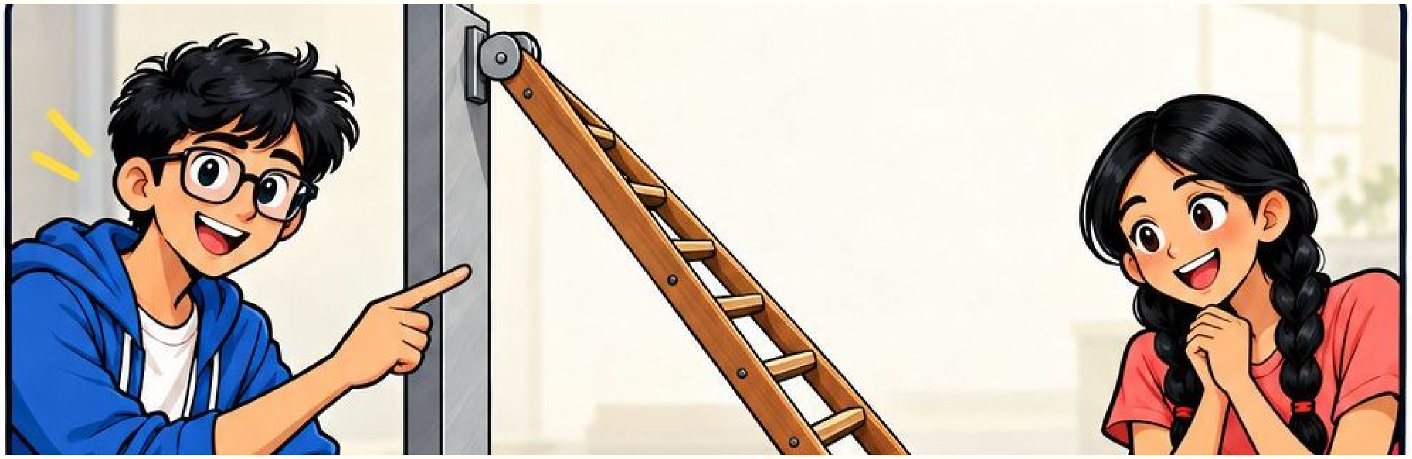
SEEMA
Correct for a frictionless wall.

PROBLEM 37 • LADDER, WALL, GROUND AND PERSON

A uniform ladder of length L and mass M rests against a frictionless vertical wall on rough ground, making angle θ to the horizontal. A person of mass m stands x from its foot. Find normal forces and minimum ground friction coefficient for equilibrium; determine maximal allowed x for given μ_s.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



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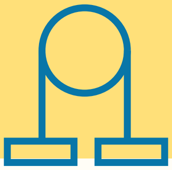


DRAW FIRST

At the frictionless wall draw only a horizontal reaction N_w; ground supplies vertical N_g and horizontal f_g.

1. Torque about the foot: $N_w L \sin\theta=[Mg(L/2)+mgx]\cos\theta$. Thus $N_w=g \cot\theta(M/2+mx/L)$.

2. Horizontal and vertical balance: $|f_g|=N_w$ and $N_g=(M+m)g$.

3. Minimum coefficient is $\mu_{min}=(M/2+mx/L)\cot\theta/(M+m)$.

4. For fixed μ_s, $x\le(L/m)[\mu_s(M+m)\tan\theta-M/2]$, additionally $0\le x\le L$. If the right side is negative, not even $x=0$ is allowed; if above L, the whole ladder length is allowed.

CHECK THE MODEL

A massive pulley has unequal tensions.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER

ROHAN
As the load moves right, can the hinge lift downward?

SEEMA
Find tension from torque, then calculate the hinge's vertical force.

ROHAN
Does its sign ever change on this beam?

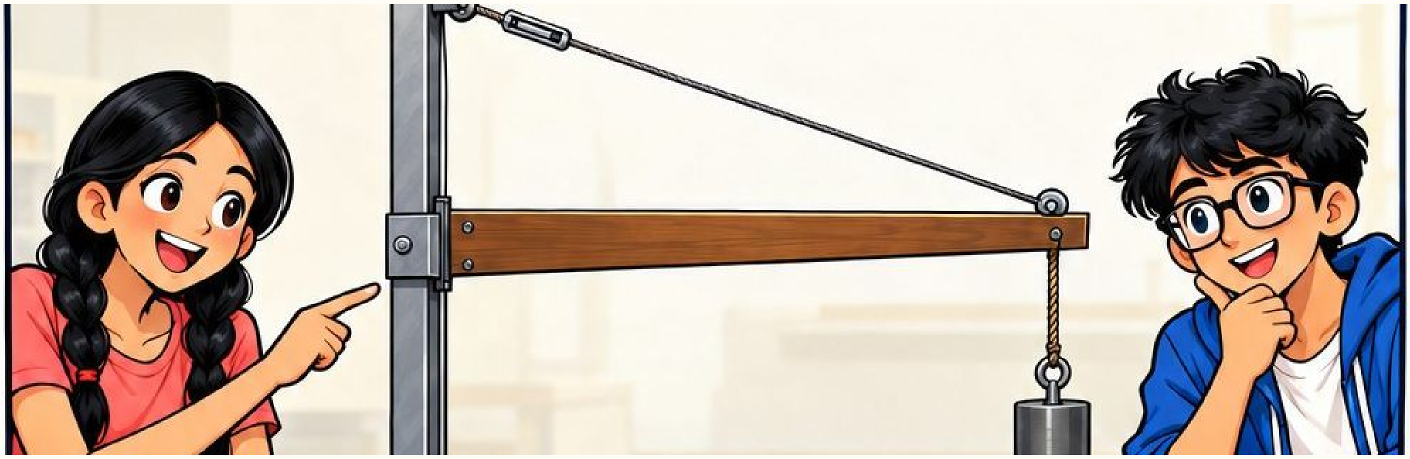
SEEMA
Evaluate the formula across $0 \le x \le L$.

PROBLEM 38 • CABLE-SUPPORTED LOADED BEAM

A horizontal uniform beam length L, mass M is hinged at left and supported at right by a cable at angle θ to the beam. A mass m hangs from position x. Determine the cable tension and hinge reaction as x varies. Find where the vertical hinge reaction changes sign, if anywhere in $0 \le x \le L$.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



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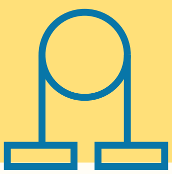


DRAW FIRST

Tension acts at $x=L$; the hinge has two signed components; beam weight at $L/2$ and hanging mass at x.

1. Moment balance at hinge: $TL\sin\theta = MgL/2 + mgx$, so $T = g(M/2 + mx/L)/\sin\theta$.
2. $H_x = -T\cos\theta$ and $H_y = (M+m)g - T\sin\theta = g[M/2 + m(1 - x/L)]$.
3. For positive M and $0 \le x \le L$, $H_y \ge Mg/2 > 0$. There is no sign change within the beam, even though the cable's vertical share grows with x.

EXAM PATTERN

Separate support forces and cable tensions.

MISCONCEPTION CHECK

A massive pulley has unequal tensions.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER

ROHAN
The table block has friction and the pulley has inertia. Start where?

SEEMA
Draw three bodies and use separate tensions.

ROHAN
Then check if the hanging block really descends?

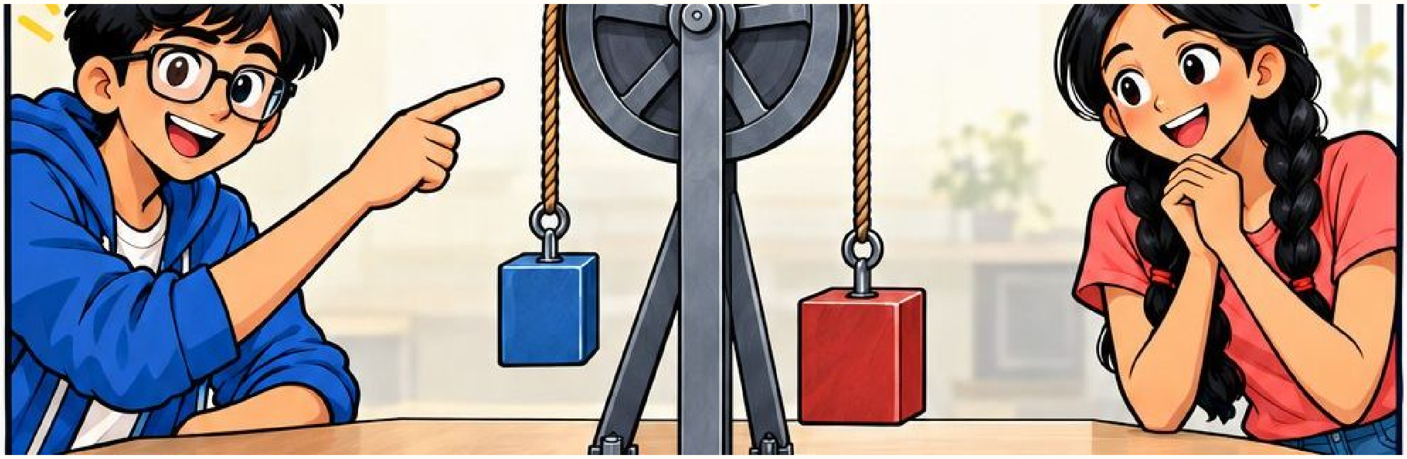
SEEMA
Yes. The numerator's sign tests that assumption.

PROBLEM 39 • BLOCK, ROUGH TABLE AND HANGING MASS

Two blocks m_1, m_2 are connected by a light inextensible string over a pulley of I, R. One block slides on a rough horizontal table with coefficient μ_k, the other hangs. Assuming motion with m_2 descending and no string slip, find a, both tensions and inequality required for that assumed direction to be dynamically consistent.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



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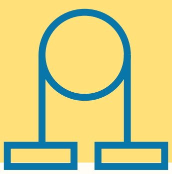


DRAW FIRST

Table block moves right; friction $\mu_k m_1 g$ left; hanging block moves down. Separate tensions T_1 and T_2.

1 $T_1 - \mu_k m_1 g = m_1 a$; $m_2 g - T_2 = m_2 a$.

2 Pulley difference $T_2 - T_1 = (I/R^2)a$. Combine: $a = g(m_2 - \mu_k m_1)/(m_1 + m_2 + I/R^2)$.

3 $T_1 = m_1(a + \mu_k g)$, $T_2 = m_2(g - a)$. The assumed downward motion is dynamically consistent for $m_2 > \mu_k m_1$; equality yields zero acceleration of an already sliding system, not necessarily a start-from-rest prediction under static friction.

EXAM PATTERN

Separate support forces and cable tensions.

MISCONCEPTION CHECK

A massive pulley has unequal tensions.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER

ROHAN
This beam touches two smooth surfaces. Why can't it rest?

SEEMA
List the directions each surface can push.

ROHAN
There is no horizontal force to balance the wall?

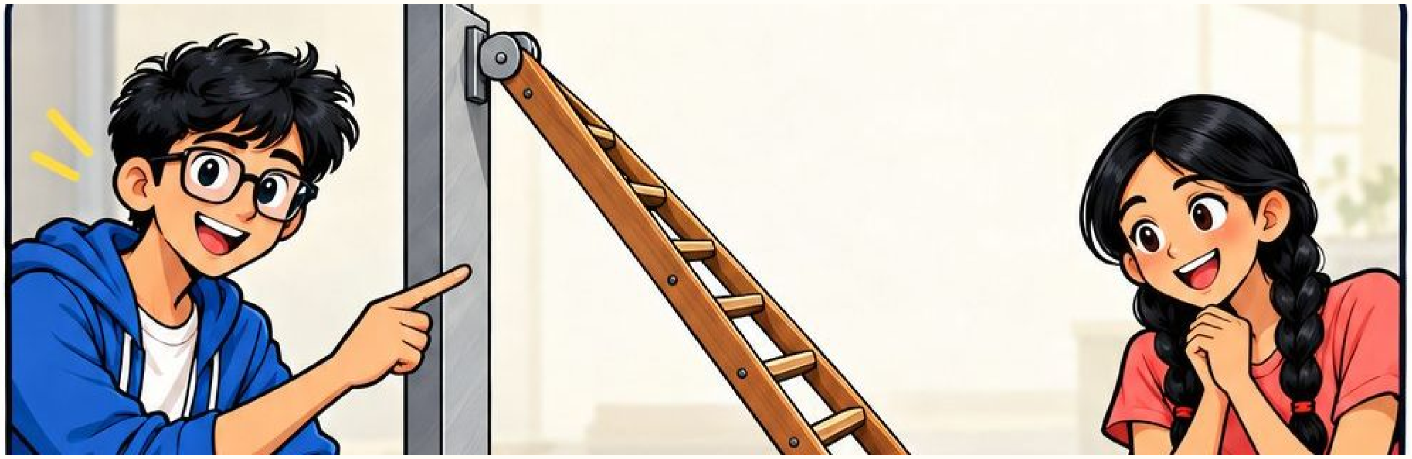
SEEMA
Exactly—and gravity's torque still needs a counter-moment.

PROBLEM 40 • BEAM BETWEEN TWO FRICTIONLESS SURFACES

A uniform beam of length L and mass M has one end on a frictionless horizontal floor and the other against a frictionless vertical wall. Show that static equilibrium at an acute angle without any other support is impossible; identify which missing horizontal force makes the torque equation inconsistent.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



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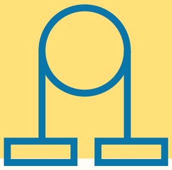


DRAW FIRST

Wall force horizontal; floor force vertical; weight downward at the beam centre. There is no horizontal counterforce.

1. Horizontal equilibrium would require $N_{wall}=0$ because the floor is frictionless.
2. About the foot, gravity has nonzero moment $Mg(L/2)\cos\theta$ for an acute angle; with $N_{wall}=0$ nothing opposes it.
3. Hence the stated static equilibrium is impossible. A horizontal floor-friction force (or another lateral support) is the missing force that permits a nonzero wall reaction.

EXAM PATTERN

Separate support forces and cable tensions.

MISCONCEPTION CHECK

A massive pulley has unequal tensions.

Model assumptions and sign conventions are stated in the worked steps.

ADVANCED MIXED REGIMES

Test every change of regime

Advanced problems change their governing conditions mid-motion: rough to smooth, collision to swing, rolling to slipping. Solve one interval at a time and carry the correct speed or angular momentum into the next.

Solve each regime separately

MISCONCEPTION

A new surface can change the motion law.

THIS CHAPTER'S PROBLEMS

41 Impact followed by an upright swing

42 Rolling sphere enters a smooth loop

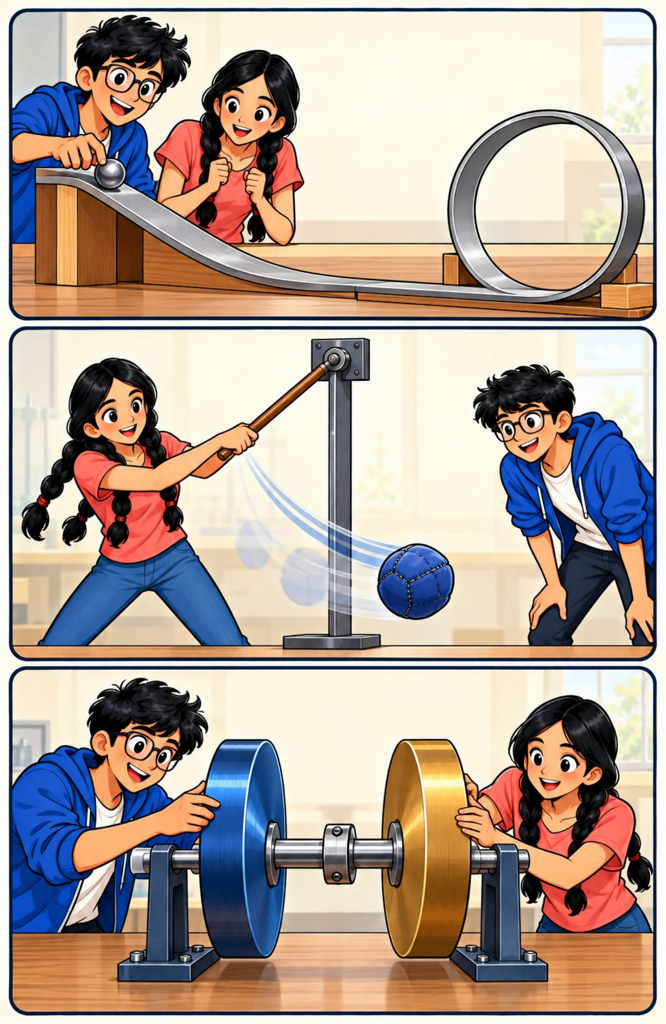
43 Coaxial discs stick by friction

44 A moving spinning rod caught at one end

45 Rough region followed by smooth floor

ADVANCED MIXED REGIMES

ROHAN & SEEMA / THINK TOGETHER



ROHAN
How fast must the particle arrive to lift the rod upright?

SEEMA
First conserve angular momentum during impact about the pin.

ROHAN
Then use energy for the upward swing?

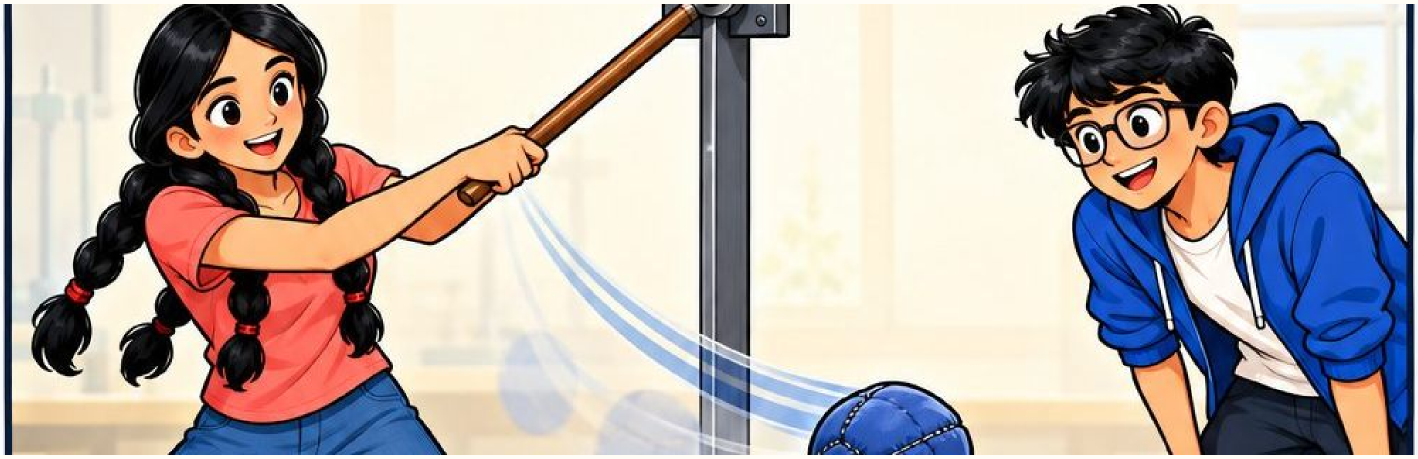
SEEMA
Yes. A rigid pin needs no taut-string top-speed rule.

PROBLEM 41 • IMPACT FOLLOWED BY AN UPRIGHT SWING

A uniform rod pivoted at one end hangs vertically at rest. A horizontal particle of mass m and speed v strikes and sticks to its free end. Find minimum v for the rod-particle system to reach the upright position; distinguish 'reach with zero speed' from completing a full revolution with a taut constraint, and state the pivot model.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



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DRAW FIRST

Draw two stages with different laws: short impact about the pin, then gravitational rise to the inverted vertical.

1. Initial rod hangs down. Transverse impact gives pivot angular momentum mvL, and after sticking $I=(M/3+m)L^2$. Thus $\omega_+=mvL/I$.

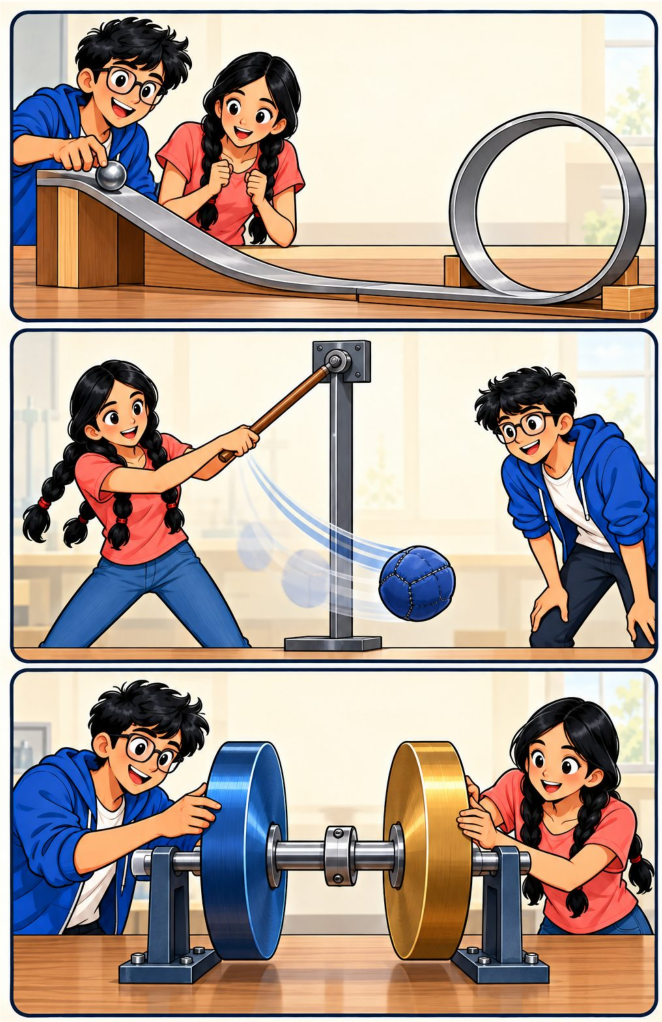
2. Rotational energy after impact is $(mvL)^2/(2I)$. To reach upright, the rod's centre rises L and the attached mass rises 2L: $\Delta U=gL(M+2m)$.

3. Equating at threshold gives $v_{min}=\sqrt{[2gL(M+2m)(M/3+m)]}/m$. At equality the assembly reaches the unstable upright position with zero speed.

4. A rigid pin can sustain the needed radial force. For a **nonzero-speed** full revolution, require $v>v_{min}$; do not import the separate taut-string condition used for a mass on a string.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER



ROHAN
The sphere rolls before the loop. Does it keep rolling inside?

SEEMA
The loop is smooth; there is no torque to change its spin.

ROHAN
So only translation energy pays for the climb?

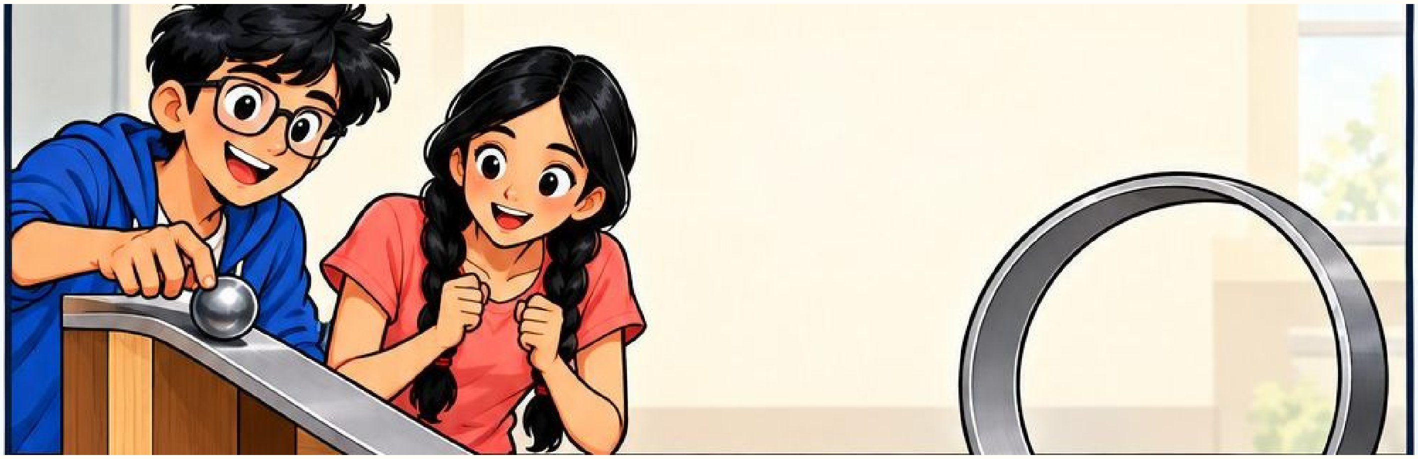
SEEMA
Exactly. At the top check the normal force is nonnegative.

PROBLEM 42 • ROLLING SPHERE ENTERS A SMOOTH LOOP

A solid sphere rolls without slipping down a height h and then enters a smooth vertical circular track of radius R measured along the sphere's centre path. Find the minimum h for maintaining contact at the top. Account for what happens to spin once the sphere enters the smooth segment.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



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DRAW FIRST

On the rough approach, spin and translation share energy; on the smooth loop, there is no tangential contact force, so spin stays constant and cannot pay for climbing.

1. At loop entry after rolling down h, $(1/2)mv_b^2(1+2/5)=mgh$, so $v_b^2=10gh/7$.

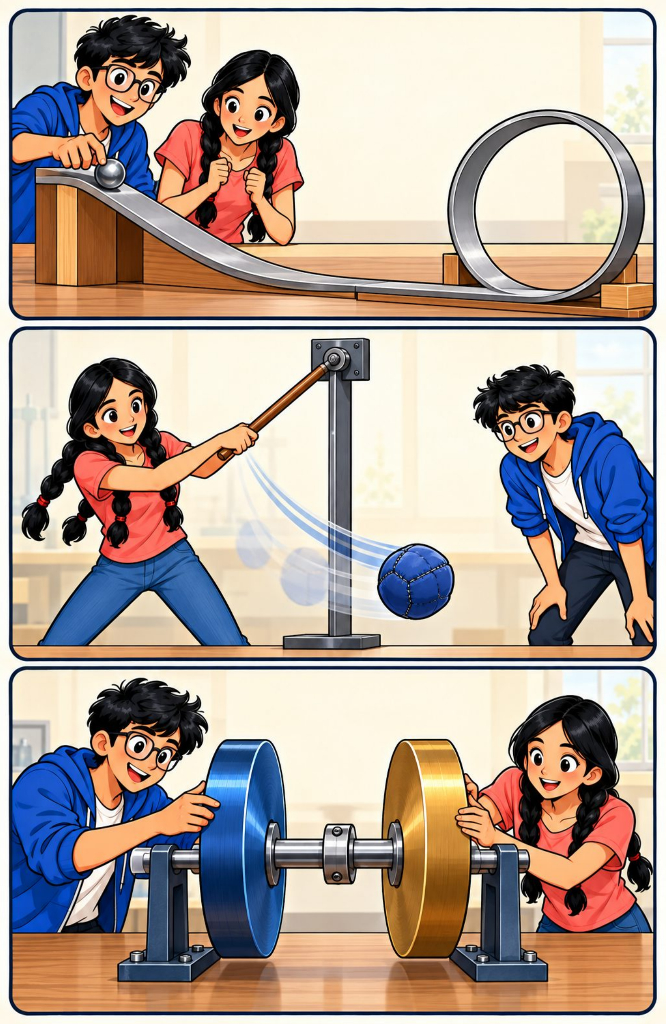
2. The loop is smooth: no torque about sphere centre from N or mg; its angular speed remains fixed. Translational energy alone changes by 2mgR between bottom and top: $v_t^2=v_b^2-4gR$.

3. At the top, inward radial equation $mg+N=mv_t^2/R$. Minimum contact has $N=0$, so $v_t^2=gR$.

4. Hence $v_b^2\geq 5gR$ and $h\geq(7/2)R$. Treat R as the **centre-path** radius. Applying rolling conservation through the smooth loop would give a wrong threshold.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER



ROHAN
The two discs lock together. Is kinetic energy conserved?

SEEMA
Friction dissipates energy, but axial angular momentum remains.

ROHAN
Their final speed is one common value?

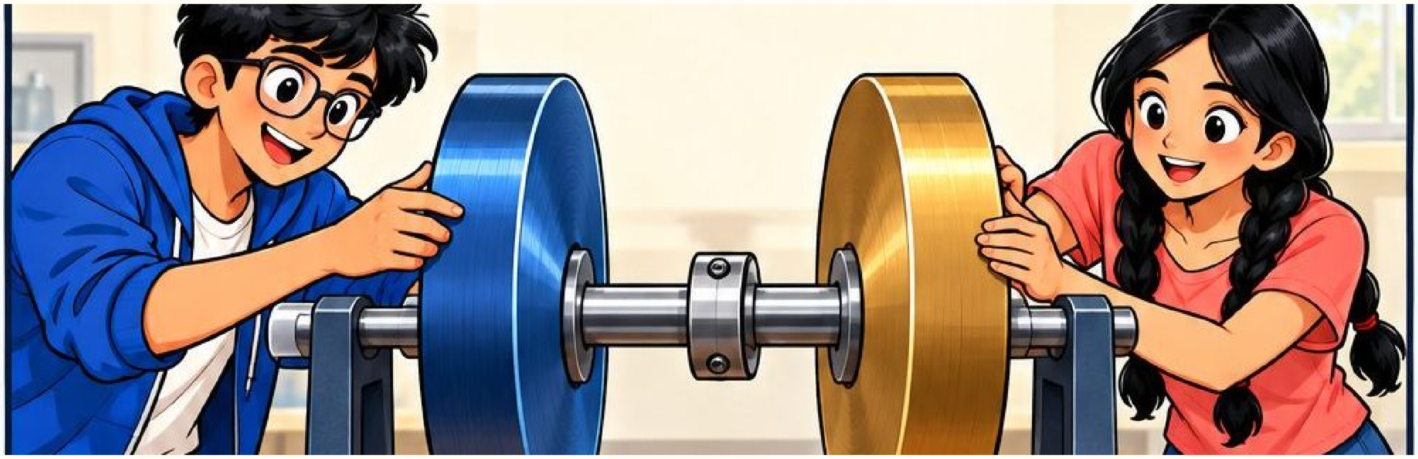
SEEMA
Yes. Compute the missing energy as heat.

PROBLEM 43 • COAXIAL DISCS STICK BY FRICTION

A spinning disc of I_1, ω_0 is placed coaxially against a second disc of I_2 initially at rest; interfacial friction locks them together, axles exert negligible torque. Find common speed and heat generated. Then evaluate the special case $I_2=3I_1$.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



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DRAW FIRST

Draw one fast disc, one stationary; friction torque is internal to the pair, so total axial L persists while kinetic energy falls.

1. $I_1\omega_0=(I_1+I_2)\omega_f$, giving $\omega_f=I_1\omega_0/(I_1+I_2)$.
2. Dissipated energy $Q=(1/2)I_1\omega_0^2-(1/2)(I_1+I_2)\omega_f^2=(I_1I_2/[2(I_1+I_2)])\omega_0^2$.
3. For $I_2=3I_1$, $\omega_f=\omega_0/4$ and $Q=(3/8)I_1\omega_0^2$.

EXAM PATTERN

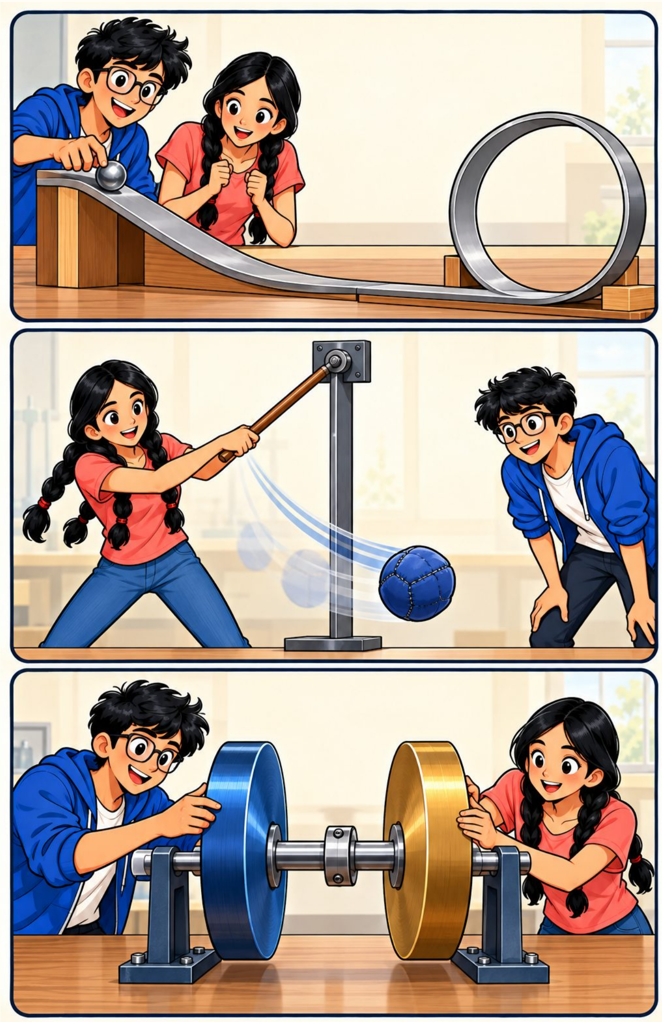
Solve piecewise and test every contact regime.

MISCONCEPTION CHECK

A new surface can change the motion law.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER



ROHAN
A moving spinning rod catches on a pin. What survives?

SEEMA
Add spin and orbital angular momentum about the new pin.

ROHAN
Could the capture lose no energy at all?

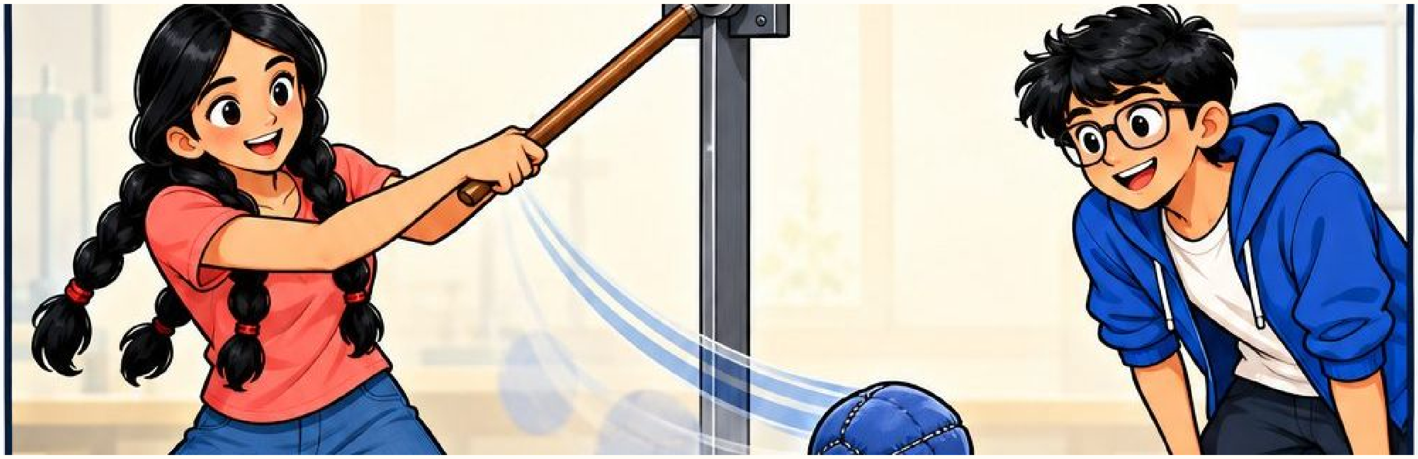
SEEMA
Only if that end already has zero velocity.

PROBLEM 44 • A MOVING SPINNING ROD CAUGHT AT ONE END

A uniform horizontal rod slides on a frictionless plane with centre speed v perpendicular to the rod toward +y and counterclockwise angular speed ω about its centre. Its LEFT end sticks instantaneously to a fixed pin when it reaches it. Determine angular speed immediately after capture and energy lost; explicitly choose the impact-time conservation axis.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



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DRAW FIRST

Pin at the left end; before capture add orbital angular momentum $Mv(L/2)$ to spin angular momentum $I_{CM}\omega$ about that future pin.

1. The pin's impact impulse has zero moment about itself. Initial $L_{pin}=(ML^2/12)\omega+MvL/2$.

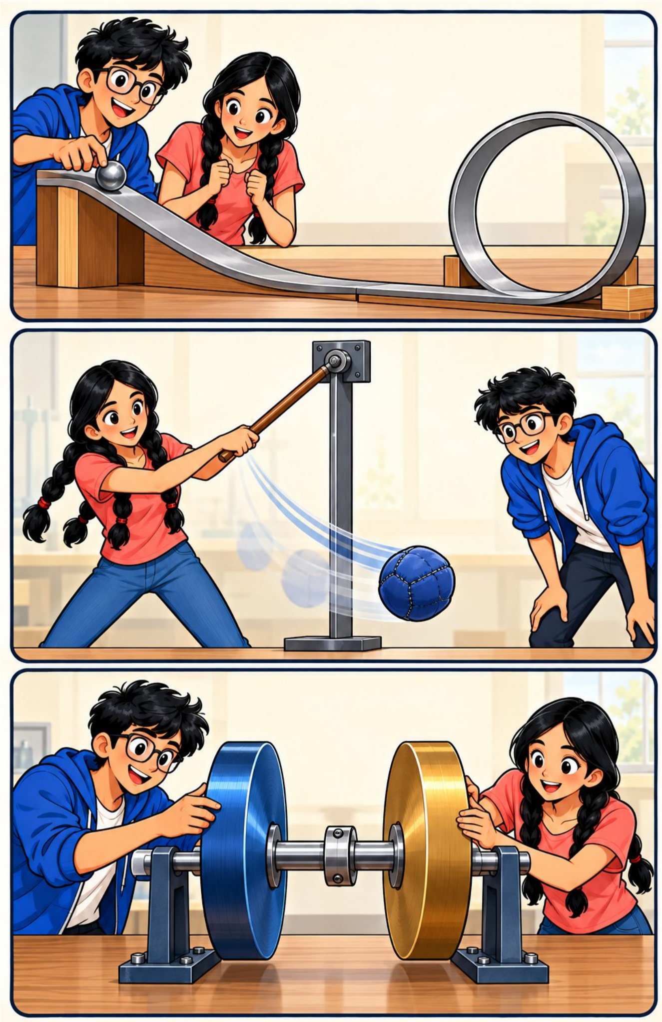
2. After capture $I_{pin}=ML^2/3$, so $\omega' = L_{pin}/I_{pin}=\omega/4+3v/(2L)$, counterclockwise for $v,\omega>0$.

3. Initial kinetic energy $K_i=(1/2)Mv^2+(ML^2\omega^2/24)$. Final $K_f=(ML^2/6)(\omega/4+3v/(2L))^2$. Loss is K_i-K_f, which is nonnegative; expanding gives $\mathbf{(M/8)(v-\omega L/2)^2}$.

4. The expression vanishes when the left endpoint already has zero velocity before capture, $v=\omega L/2$. This is a strong physical check.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER



ROHAN
Will the cylinder be rolling before the rough strip ends?

SEEMA
Find when its bottom slip first reaches zero.

ROHAN
What if the smooth region starts earlier?

SEEMA
Then its current translation and spin each remain unchanged.

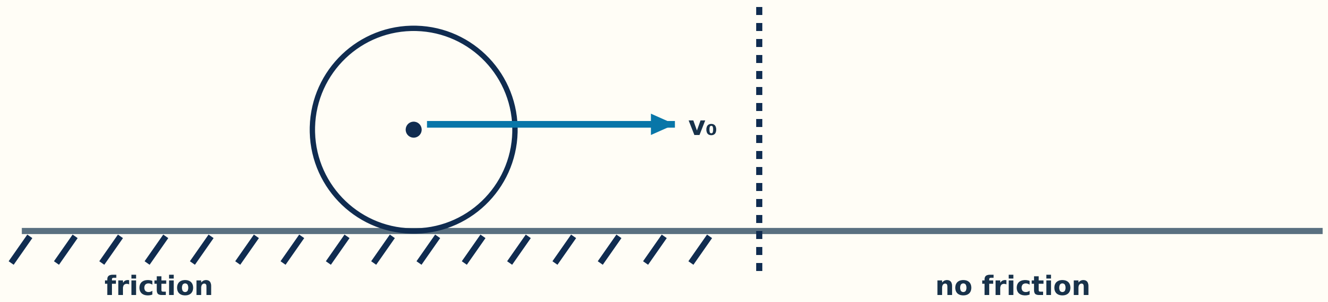
PROBLEM 45 • ROUGH REGION FOLLOWED BY SMOOTH FLOOR

A solid cylinder with initial centre speed $v_0>0$ rightward and signed clockwise angular speed ω_0 encounters a rough floor followed by a smooth floor after a distance D to its right. Assume its centre stays right-moving until pure rolling begins. Determine the threshold D needed to attain pure rolling before reaching the boundary, and describe the subsequent motion on the smooth section for each regime of initial slip.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.

ROUGH FLOOR → SMOOTH FLOOR



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DRAW FIRST

Draw initial slip $u_0=v_0-R\omega_0$, rolling point x_r, and the boundary at $x=D$. Use clockwise ω positive.

1 From Q28, $t_r=|u_0|/(3\mu g)$ and $x_r=v_0t_r-(\mu g/2)\mathrm{sgn}(u_0)t_r^2$. Under the stated right-moving assumption, the threshold is $D^*=x_r$.

2 If $D\geq D^*$, the cylinder enters the smooth region in pure rolling with $v_r=(2v_0+R\omega_0)/3$ and $\omega_r=v_r/R$; both remain constant there.

3 If $0\leq D<D^*$, it is still slipping at the boundary. Find t_D from $D=v_0t_D-(\mu g/2)\mathrm{sgn}(u_0)t_D^2$, taking the first root $0\leq t_D<t_r$. Entry values are $v_D=v_0-\mu g\,\mathrm{sgn}(u_0)t_D$ and $\omega_D=\omega_0+(2\mu g/R)\mathrm{sgn}(u_0)t_D$.

4 On the smooth section both v_D and ω_D remain constant separately, so its slip persists. The smooth surface cannot create the torque needed to finish synchronizing them.

Model assumptions and sign conventions are stated in the worked steps.

EXAM LABORATORY

Decide, solve, and verify

Select the method from the time scale and forces: dynamics for acceleration, energy over a displacement, angular momentum during a short impact. Finish with dimensions, signs, limiting cases and contact validity.

Model → law → answer → check

MISCONCEPTION

An answer needs a validity check.

THIS CHAPTER’S PROBLEMS

46 Multiple-correct rolling claims

47 Numerical cylinder check

48 Conservation during a pinned-rod impact

49 Upper versus lower inner-axle tangent

50 Two independent pulley methods

ROHAN & SEEMA / THINK TOGETHER

ROHAN
The contact point is at rest. Is its acceleration zero too?

SEEMA
Instantaneous zero velocity does not imply zero acceleration.

ROHAN
Do we always need friction to keep rolling?

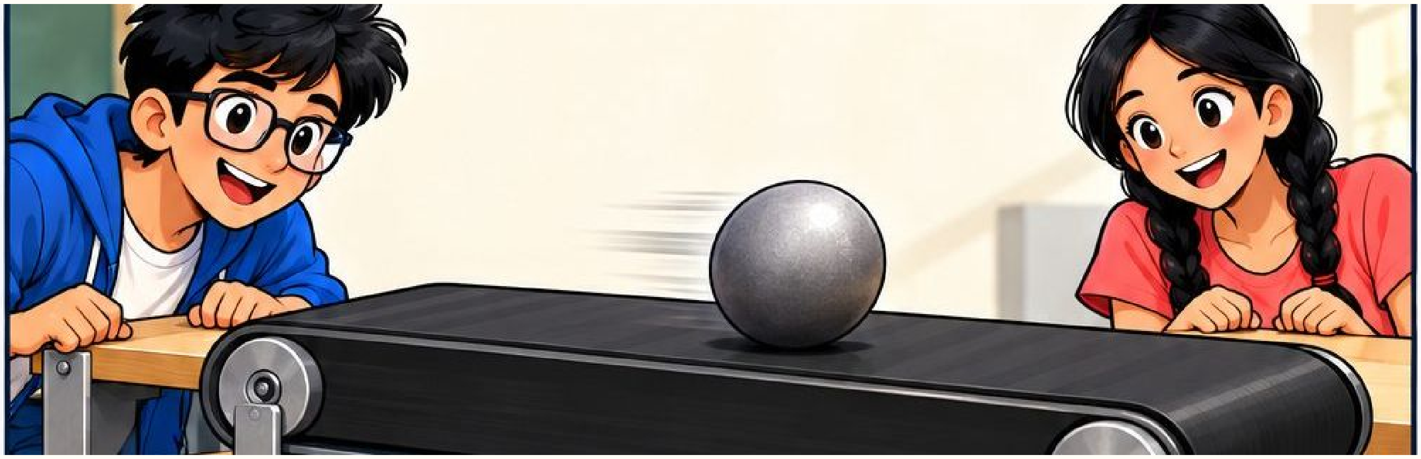
SEEMA
No. Check each statement against a constant-speed wheel.

PROBLEM 46 • MULTIPLE-CORRECT ROLLING CLAIMS

Multiple correct: A rigid body rolls without slipping on a stationary horizontal surface. Evaluate: (A) contact point has zero instantaneous velocity in the ground frame; (B) contact point has zero acceleration; (C) static friction must be nonzero; (D) total kinetic energy can be written $\frac{1}{2}I_{contact}\,\omega^2$. Justify all options and state the axis theorem used.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



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DRAW FIRST

Mark the bottom contact as instantaneously stationary, but draw its acceleration and the centre's acceleration separately.

1. **A true:** no slip on stationary ground gives $v_{contact}=0$ at that instant.

2. **B false:** zero velocity does not imply zero acceleration; e.g. a rim point on a uniformly rolling wheel has acceleration toward the centre.

3. **C false:** a wheel can roll at constant speed on level ground with no required friction force.

4. **D true:** $K=(1/2)Mv_{CM}^2+(1/2)I_{CM}\omega^2=(1/2)(I_{CM}+MR^2)\omega^2=(1/2)I_{contact}\omega^2$ by the parallel-axis theorem. Answer: **A, D**. The contact is an instantaneous velocity axis, not automatically an axis usable for every torque derivative.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER

ROHAN
Can we solve this cylinder with forces and with energy?

SEEMA
Use translation and torque for acceleration and friction.

ROHAN
Then check v^2 from the vertical drop?

SEEMA
Yes. Two methods should agree on the final speed.

PROBLEM 47 • NUMERICAL CYLINDER CHECK

Numerical answer: A uniform solid cylinder rolls down a 30° fixed rough incline through vertical height $h=1.5$ m, starting from rest. Use $g=10$ m/s². Find v^2 at bottom in m²/s², acceleration in m/s² and minimum μ_s; report exact expressions where needed.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.

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DRAW FIRST

Separate the downhill force balance from rotational torque, then cross-check by energy over the vertical drop.

1. For a solid cylinder $k=1/2$, $a=g\sin 30°/(1+k)=10/3$ m/s².
2. $f=[k/(1+k)]mg\sin 30°=mg/6$; with $N=mg\cos 30°$, $\mu_{min}=f/N=1/(3\sqrt{3})$.
3. Energy gives $v^2=2gh/(1+k)=2(10)(1.5)/(1.5)=20$ m²/s².
4. Geometric check: path length $s=h/\sin 30°=3$ m and $2as=2(10/3)(3)=20$. Results: **20 m²/s², 10/3 m/s², 1/(3√3)**.

CHECK THE MODEL

An answer needs a validity check.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER

ROHAN
Which quantity survives when a particle sticks to a pinned rod?

SEEMA
Inspect the pin impulse about each proposed axis.

ROHAN
It has no moment only about the pin itself?

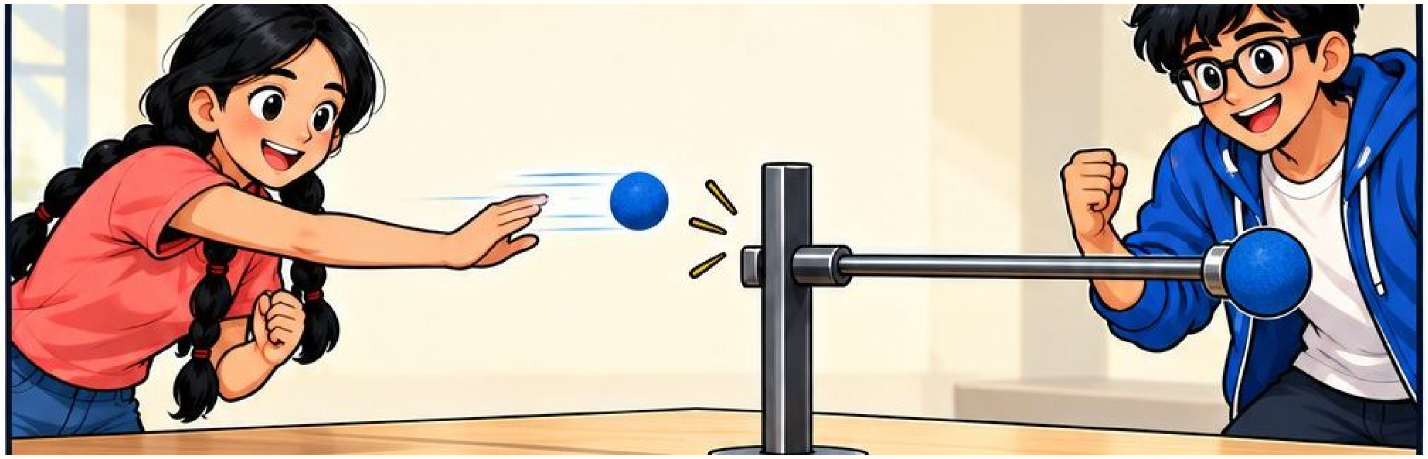
SEEMA
Correct. Sticking also destroys mechanical energy.

PROBLEM 48 • CONSERVATION DURING A PINNED-ROD IMPACT

Multi-select: A particle strikes a rod pivoted at one end and sticks. During the short impact, assess which quantities can be conserved for the particle-plus-rod system: linear momentum, angular momentum about pivot, angular momentum about rod centre, mechanical energy. State the impulse assumptions underlying each choice.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



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DRAW FIRST

Draw the external pin impulse at the pivot; its moment is zero about the pivot but generally nonzero about the rod centre.

1. Linear momentum of particle+rod is generally **not conserved** because the pin supplies an external impulse.

2. Angular momentum about the pivot **is conserved during the short impact** if external gravity's impulse and axle friction torque are negligible; the pin impulse has no lever arm there.

3. Angular momentum about the rod centre is **not generally conserved** because the pin impulse has a lever arm about that centre. Mechanical energy is not conserved for sticking.

4. The conclusion depends on the short-duration impulse model; a substantial external torque about the pivot would invalidate item 2.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER

ROHAN
Why do the two string positions move the spool differently?

SEEMA
Measure each force's lever arm from the floor contact.

ROHAN
Can the upper case need no friction at all?

SEEMA
For these particular I and r values, yes. Verify it.

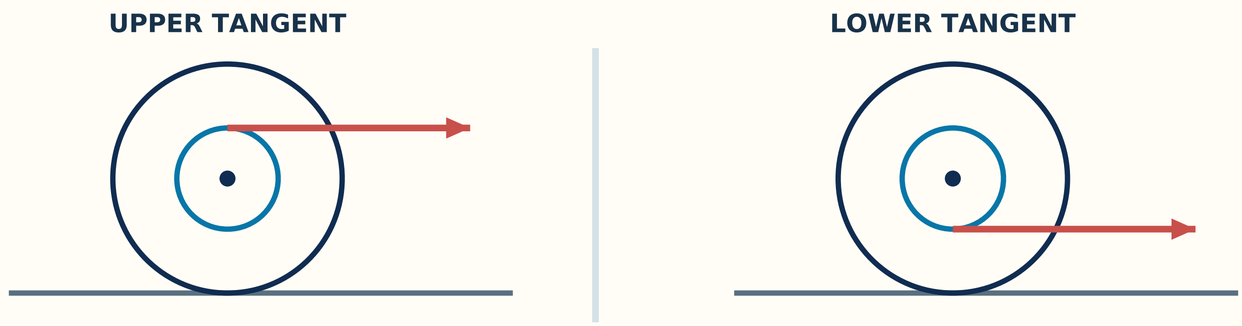
PROBLEM 49 • UPPER VERSUS LOWER INNER-AXLE TANGENT

Linked comprehension: A spool with outer radius R and inner radius R/2 rolls without slipping on level ground. A horizontal force F pulls a string tangent to the top of its inner axle. Given $I=MR^2/2$, find the translational acceleration and friction force; then move the rightward pull to a string tangent to the BOTTOM of the inner axle and re-evaluate. Include a signed diagram for each case.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.

SPOOL CROSS-SECTIONS / RIGHTWARD PULL



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DRAW FIRST

Draw the outer floor contact and two horizontal force lines at heights $R+r$ and $R-r$ above it, with $r=R/2$.

1. Effective rolling inertia about floor contact is $I+MR^2=(3/2)MR^2$. For a horizontal rightward pull at the **upper** inner-axle tangent, $a=FR(R+r)/(I+MR^2)=F/M$.

2. Cylinder translation $F+f=Ma$ gives $f=0$ for this particular I and r. Angular acceleration is clockwise $a/R=F/(MR)$.

3. Move the same rightward pull to the **lower** inner-axle tangent: $a=FR(R-r)/(I+MR^2)=F/(3M)$. Hence $f=Ma-F=-2F/3$, leftward on the spool, and clockwise $\alpha=F/(3MR)$.

4. The zero upper-tangent friction is a special parameter result, not a general feature of spools. Confirm sufficient static friction for the lower case: $\mu_s Mg \geq 2F/3$.

Model assumptions and sign conventions are stated in the worked steps.

ROHAN & SEEMA / THINK TOGETHER

ROHAN
Can one pulley problem have two independent solutions?

SEEMA
Use two tensions and torque, then check with energy.

ROHAN
What mistake would equal tensions cause?

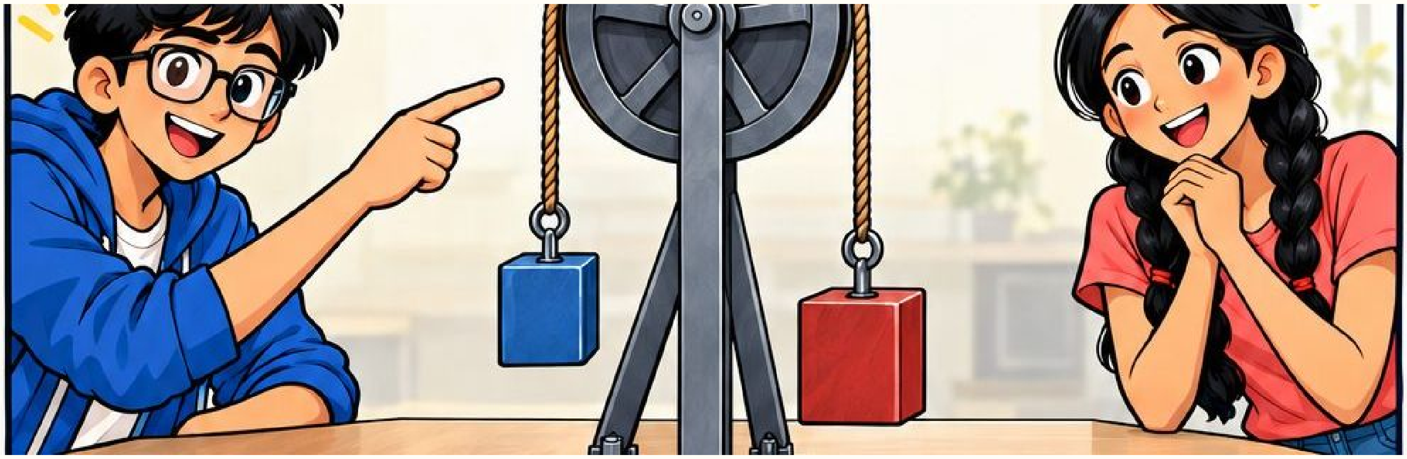
SEEMA
A massive pulley would have zero torque despite accelerating.

PROBLEM 50 • TWO INDEPENDENT PULLEY METHODS

Open-ended verification: Design a two-method solution for a massive pulley and two hanging masses: first Newton-plus-torque, then energy-plus-constraint. Show their acceleration formulas agree, check the zero-pulley-inertia limit, and identify one plausible but wrong use of equal tensions that each method would expose.

YOUR FIRST EQUATION

Predict the method before turning to the worked solution.



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DRAW FIRST

Newton method uses two tensions and a pulley torque; energy method uses one shared displacement x and the rotational speed v/R.

1. Newton: $m_2g-T_2=m_2a$; $T_1-m_1g=m_1a$; $(T_2-T_1)R=I\,a/R$. Thus $a=(m_2-m_1)g/(m_1+m_2+I/R^2)$.

2. Energy over a small descent dx of m_2: net gravity work $(m_2-m_1)g\,dx$ increases kinetic energy of both blocks and pulley. Differentiate $[(1/2)(m_1+m_2+I/R^2)v^2]$ with respect to x, using $a=v\,dv/dx$, to get the same formula.

3. With $I\to0$ both methods give $a=(m_2-m_1)g/(m_1+m_2)$. Assuming $T_1=T_2$ prematurely makes pulley torque zero and would force $\alpha=0$ for finite I, contradicting no slip when $a\neq0$.

CHECK THE MODEL

An answer needs a validity check.

Model assumptions and sign conventions are stated in the worked steps.

THE ROTATIONAL-MOTION CHECKLIST

1. Pick a fixed pivot or valid instantaneous axis.

2. Keep the force diagram separate from its resolved components.

3. Use no-slip relative to the surface, not automatically to ground.

4. Conserve angular momentum only when external torque about the chosen axis is negligible.

5. Test string tension, static friction, normal force, direction and limiting cases.

OBSERVE → MODEL → SOLVE → VERIFY